BEEM103 – Optimization Techiniques for Economists	Dieter Balkenborg Departments of Economics
Homework Week 3	University of Exeter

Exercise 1 Show that the function

$$Q = K^{\alpha} L^{\beta}$$

with $\alpha, \beta > 0$ and $\alpha + \beta < 1$ is concave.

Exercise 2 Use the least-square criterion to find the equation of the line that is closest to the points $(1,1)$, $(2,3)$ and $(4,3)$. That is, find the line $y = f(x) = ax + b$ which minimizes

$$F(a,b) = \sum_{i=1}^{n} (y_i - f(x_i))^2$$

where the summation is over the above three data points (x_i, y_i).

Exercise 3 The highway department is planning to build a picnic area for motorists along a major highway. It is to be rectangular with an area of $5,000$ square yards and is to be fenced off on the three sides not adjacent to the highway. What is the least amount of fencing that will be needed to complete the job?

a) Identify this problem as a constraint optimization problem. What objective function $f(x,y)$ is to be maximized / minimized subject to what constraint $g(x,y) \geq 0$?

b) Write down the Lagrangian $\mathcal{L}(x,y)$ for this problem.

c) Find the solution to the three equations a) $\frac{\partial \mathcal{L}}{\partial x} = 0$ b) $\frac{\partial \mathcal{L}}{\partial y} = 0$ c) $g(x,y) \geq 0$.