BEEM103 – Optimization Techiniques for Economists	Dieter Balkenborg Departments of Economics
Homework Week 2	University of Exeter

Exercise 1 Calculate the first and second order partial derivatives of

$$\begin{aligned} \text{a) } z &= (2-x-y)\,x + \left(5+2x-3y^2\right)y - 3x + 2y^2 \\ \text{b) } z &= \left(x^2+y^3\right)^5 \end{aligned}$$

Exercise 2 Find the critical point of the functions

$$\begin{aligned} \text{a) } z &= f(x,y) = xy - 2x + 3y - 6 \\ \text{b) } z &= g(x,y) = 2x^2 + 2xy - 6x + 5y^2 - 6y + 5 \\ \text{c) } z &= h(x,y) = -2x^2 - 2xy + 6x - 5y^2 - 5 \end{aligned}$$

Determine whether they are troughs, peaks or saddlepoints.

Exercise 3 A dairy produces whole milk and skim milk in quantities x and y gallons, respectively. Suppose that the price of whole milk is $p(x) = 20 - 5x$ and that the price of skim milk is $q(y) = 4 - 2y$ and assume that $C(x,y) = 2xy + 4$ is the total (!) joint-cost function of the commodities. What should x and y be to maximize profit, assuming that the first order conditions yield a maximum? Show that your solution is indeed the unique maximum.