Exercise 1 Bring onto common denominator and simplify
\[
\frac{2t - t^2}{2t + 2} \left( \frac{5t}{t - 2} - \frac{2t}{t - 2} \right)
\]

Exercise 2 Bring onto common denominator and simplify
\[
\frac{\frac{a}{x} - \frac{a}{y}}{\frac{a}{x} + \frac{a}{y}}
\]

Exercise 3 Simplify
\[
\frac{8\sqrt{x^2} \sqrt{y} \sqrt{1/z}}{-2\sqrt{x} \sqrt{y^5} \sqrt{z}} \left( (3a)^{-1} - 2 (2a^{-2})^{-1} \right)/a^{-3}
\]

Exercise 4 Solve
\[
\frac{3}{x - 3} - \frac{2}{x + 3} = \frac{9}{x^2 - 9}
\]

Exercise 5 Solve
\[
K^{1/2} \left( \frac{1}{2} \frac{r}{w} K \right)^{1/4} = Q \quad \text{for } K
\]
\[
[(1 - \lambda) a^{-\rho} + \lambda b^{-\rho}] = c \quad \text{for } b
\]

Exercise 6 Expand and simplify
\[
(1 + q + q^2 + \cdots + q^n) (1 - q)
\]

Exercise 7 Simplify
\[
\exp(\ln(x)) - \ln(\exp(x))
\]
solve
\[
\ln x^{5/2} - 0.5 \ln x = \ln 25
\]

Exercise 8 Find the derivative of
\[
(2x + 1)^{10} \ln(1 - x^3)
\]

Exercise 9 Sketch the graph of a function \( y(x) \) that has all the following properties:
i) \( y'(x) > 0 \) when \( x < 0 \) and when \( x > 4 \)
ii) \( y'(x) < 0 \) when \( 0 < x < 4 \)
iii) \( y''(x) > 0 \) when \( x > 3 \)
iv) \( y''(x) < 0 \) when \( x < 3 \).
Exercise 10 Consider the function

\[ y(x) = e^{-x^2} \]

i) Calculate and draw a sign diagram for the first derivative. Where is the function increasing or decreasing. Are there any peaks or troughs? Does the function have an (absolute) maximum.

ii) Calculate and draw a sign diagram for the second derivative. Where is the function convex or concave. Are there any inflection points?

Exercise 11 Find all critical points of the function

\[ y = \frac{1}{4}x^4 - 2x^2 \]

Are these local maxima or minima?

Exercise 12 For the function

\[ y = \frac{1}{4}x^4 - 2x^2 \]

find the maximum a) on the interval \([-2, 2]\) and b) on the interval \([-4, 4]\).

Exercise 13 A producer operating in a perfectly competitive market has the total cost function

\[ TC(Q) = 2Q^3 - 18Q^2 + 60Q + 50 \]

where costs are given in Pounds Sterling.

1. Calculate and sketch the marginal cost function \(MC(Q)\) and the average variable cost function \(AVC(Q)\).
2. Solve the equation \(MC(Q) = AVC(Q)\).
3. At what quantity are average variable costs minimized?
4. What are the minimum average variable costs?
5. What quantity maximizes profits when the market price is \(P = 10\)?
6. What is the maximal profit the firm can make at this price?
7. What quantity maximizes profits when the market price is \(P = 25\)?