BEEM103 – Optimization Techiniques for Economists	Dieter Balkenborg Departments of Economics
Homework Week 1	University of Exeter

Exercise 1 Bring onto common denominator and simplify

$$\frac{2t-t^2}{2t+2}\left(\frac{5t}{t-2}-\frac{2t}{t-2}\right)$$

Exercise 2 Bring onto common denominator and simplify

$$\frac{\frac{a}{x}-\frac{a}{y}}{\frac{a}{x}+\frac{a}{y}}$$

Exercise 3 Simplify

$$\frac{8\sqrt[3]{x^2}\sqrt[4]{y}\sqrt{1/z}}{-2\sqrt[3]{x}\sqrt{y^5}\sqrt{z}} \qquad \left(\left((3a)^{-1}\right)^{-2}\left(2a^{-2}\right)^{-1}\right)/a^{-3}$$

Exercise 4 Solve

$$\frac{3}{x-3}-\frac{2}{x+3}=\frac{9}{x^2-9}$$

Exercise 5 Solve

$$\begin{aligned} K^{1/2}\left(\frac{1}{2}\frac{r}{w}K\right)^{1/4} &= Q \qquad \text{for } K \\ \left[(1-\lambda)\,a^{-\rho}+\lambda b^{-\rho}\right] &= c \qquad \text{for } b \end{aligned}$$

Exercise 6 Expand and simplify

$$\left(1+q+q^2+\cdots+q^n\right)(1-q)$$

Exercise 7 Simplify

$$\exp(\ln(x))-\ln(\exp(x))$$

solve

$$\ln x^{5/2}-0.5\ln x=\ln 25$$

Exercise 8 Find the derivative of

$$(2x+1)^{10}$$
$$\ln\left(1-x^3\right)$$

Exercise 9 Sketch the graph of a function $y(x)$ that has all the following properties:

i) $y'(x)>0$ when $x<0$ and when $x>4$
ii) $y'(x)<0$ when $0<x<4$
iii) $y''(x)>0$ when $x>3$
iv) $y''(x)<0$ when $x<3$.

Exercise 10 Consider the function

$$y(x) = e^{-x^2}$$

i) Calculate and draw a sign diagram for the first derivative. Where is the function increasing or decreasing. Are there any peaks or troughs? Does the function have an (absolute) maximum.

ii) Calculate and draw a sign diagram for the second derivative. Where is the function convex or concave. Are there any inflection points?

Exercise 11 Find all critical points of the function

$$y = \frac{1}{4}x^4 - 2x^2$$

Are these local maxima or minima?

Exercise 12 For the function

$$y = \frac{1}{4}x^4 - 2x^2$$

find the maximum a) on the interval $[-2, 2]$ and b) on the interval $[-4, 4]$.

Exercise 13 A producer operating in a perfectly competitive market has the total cost function

$$TC(Q) = 2Q^3 - 18Q^2 + 60Q + 50$$

where costs are given in Pounds Sterling.

1. Calculate and sketch the marginal cost function $MC(Q)$ and the average variable cost function $AVC(Q)$.
2. Solve the equation $MC(Q) = AVC(Q)$.
3. At what quantity are average variable costs minimized?
4. What are the minimum average variable costs?
5. What quantity maximizes profits when the market price is $P = 10$?
6. What is the maximal profit the firm can make at this price?
7. What quantity maximizes profits when the market price is $P = 25$?