

BEEM103 – Optimization Techniques for Economists	Dieter Balkenborg Departments of Economics
Class Exercises Week 6	University of Exeter

Exercise 1 Find the solutions of the form $e^{\alpha t}$ to the homogenous equation

$$\ddot{x} - 2\dot{x} + x = 0$$

Exercise 2 Find a solution to the inhomogeneous equation

$$\ddot{x} - 2\dot{x} + x = 3$$

Exercise 3 Describe all solutions to this equation.

Exercise 4 Find a solution with $x(0) = 0$, $x(1) = 1$

Exercise 5 Find the solutions to the homogenous equation

$$\ddot{x} + x = 0$$

Exercise 6 Minimize

$$\int_0^1 \sqrt{1 + \dot{x}^2} dt$$

subject to the boundary conditions $x(0) = 4$, $x(1) = 2$.

Exercise 7 Minimize

$$\int_0^1 (x^2 + 2tx\dot{x} + t^2\dot{x}^2) dt$$

subject to the boundary conditions $x(0) = 0$, $x(1) = 2$.

Exercise 8 Find the Euler equation for the Ramsey problem

$$\int_0^T U \left(f(K(t)) - \dot{K}(t) \right) e^{-rt} dt$$

subject to the boundary conditions $x(0) = 4$, $x(1) = 2$. Use the relative rate of change consumption \dot{C}/C where $C = f(K) - \dot{K}$.

Exercise 9 Solve the equation in the case $f(K) = bK$, $U(C) = C^{1-v}/(1-v)$.