BEEM103 – Optimization Techiniques for Economists	Dieter Balkenborg Departments of Economics
Class Exercises Week 3	University of Exeter

Exercise 1 A monopolist produces two commodities in quantities Q_1 and Q_2. The prices for the two commodities are P_1 and P_2, respectively. Demand for each commodity is given by

$$Q_1 = 50 - 2P_1 + P_2 \qquad Q_2 = 75 + 2P_1 - 3P_2$$

Total costs for producing Q_1 (respectively Q_2) units of the first (second) commodity are $15Q_1$ and $10Q_2$, respectively.

1 Express the total revenue from sales of the two commodities in terms of the variables Q_1, Q_2, P_1, P_2. Then, using the demand functions express total revenue as a function $TR(P_1, P_2)$ of the prices only.

2 Assuming that he sells what he is producing, express total production costs as a function $TC(P_1, P_2)$ of the prices only.

3 Find a critical point of the profit function.

4 Use second partial derivatives and the Hessian matrix to show that the critical point is a relative profit maximum.

Exercise 2 A consumer with utility function $u(x,y) = (x+1)(y+1)$ has a budget $b = 100$, while the prices are $p_x = 20$, $p_y = 40$. Write down the Lagrangian for the problem and find its partial derivatives. Assume that only the budget constraint is binding. Which Lagrange multipliers must then be zero? What system of equations must be satisfied and what is its solution? Is the solution reasonable?

Exercise 3 A consumer with utility function $u(x,y) = (x+1)(y+1)$ has a budget $b = 44$, while the prices are $p_x = 100$, $p_y = 16$. Write down the Lagrangian for the problem and find its partial derivatives. Assume that only the budget constraint is binding. Which Lagrange multipliers must be then be zero? What system of equations must be satisfied and what is its solution? Is the solution reasonable?

Exercise 4 What is the consumer optimum in the last exercise?

Exercise 5 The highway department is planning to build a picnic area for motorists along a major highway. It is to be rectangular with an area of $5,000$ square yards and is to be fenced off on the three sides not adjacent to the highway. What is the least amount of fencing that will be needed to complete the job?

a) Identify this problem as a constraint optimization problem. What objective function $f(x,y)$ is to be maximized / minimized subject to what constraint $g(x,y) \geq 0$?

b) Write down the Lagrangian $\mathcal{L}(x,y)$ for this problem.

c) Find the solution to the three equations a) $\frac{\partial \mathcal{L}}{\partial x} = 0$ b) $\frac{\partial \mathcal{L}}{\partial y} = 0$ c) $g(x,y) \geq 0$.

Exercise 6 A consumer has the following utility when he consumes x units of apples and y units of oranges:

$$u(x, y) = -x^2 + 4x - y^2 + 16y$$

Suppose the consumer has a budget of £3.20 to be spend on oranges and apples. Each apple and each orange costs £0.40. Use the method of Lagrange to find the optimal consumption bundle:

a) Write down the budget constraint and the Lagrangian. Assume that only the budget constraint is binding.