BEEM103 – Optimization Techiniques for Economists	Dieter Balkenborg Departments of Economics
Class Exercises Week 2	University of Exeter

Exercise 1 Calculate the partial derivatives of

$$\begin{aligned} \text{a) } z &= 5y^5 + 4x^4y + 3x^2y^3 + 2xy^4 + 2x + 3y + 5 \\ \text{b) } z &= \frac{xy^2}{x^2y^3 + 1} \\ \text{c) } z &= \left(x^9y + 1\right)\left(xy^8 + 1\right) \end{aligned}$$

Exercise 2 Find all second derivatives $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial y \partial x}$, $\frac{\partial^2 z}{\partial x \partial y}$ and $\frac{\partial^2 z}{\partial y^2}$ of

$$z = 5x^2y + 3x^2y^2 + 5y^3$$

Exercise 3 Find the equations $z = ax + c$ for the tangents to the graph of the function

$$z = f(x) = x^3 - 4x$$

for $x_0 = 1$ and for $x_0 = 3$.

Hint: Use the formula

$$dz = f'(x_0)\, dx$$

where $dx = x - x_0$, $dz = z - z_0$ and $z_0 = f(x_0)$.

Exercise 4 Find the equations $z = ax + by + c$ for the tangent planes to the graph of the function

$$z = f(x, y) = x^3y - 4xy^2$$

for $(x_0, y_0) = (1, 2)$ and for $(x_0, y_0) = (2, 1)$.

Hint: Use the formula

$$dz = \frac{\partial z}{\partial x}_{|x=x_0, y=y_0} dx + \frac{\partial z}{\partial y}_{|x=x_0, y=y_0} dy$$

where $dx = x - x_0$, $dy = y - y_0$, $dz = z - z_0$ and $z_0 = f(x_0)$.

Exercise 5 For the function

$$z = xy$$

find the slope of the level curves in the points $(1, 2)$ and $(2, 2)$. Find the equations for the tangents.

Exercise 6 Suppose a consumer has the utility function $u(x, y) = xy$ and his budget is 40. Given your knowledge of economics from economics and the results from the previous question, what will be his demand when the prices are a) $p_x = p_y = 10$ and b) $p_x = 40$, $p_y = 10$?

Exercise 7 Find the critical point of the function

$$z = 20x^2 - 37xy + 31x + 15y^2 - 16y - 7$$

Is it a maximum, a minimum or a saddle point?

Exercise 8 A T-shirt shop carries two competing shirts, one endorsed by Michael Jordan and the other by Shaq O'Neal. The owner of the store can obtain both at a cost of \$2 per shirt and estimates that if Jordan shirts are sold for x dollars apiece and O'Neal shirts for y dollars apiece, consumers will buy approximately $40 - 50x + 40y$ Jordan shirts and $20 + 60x - 70y$ O'Neil shirts each day.

a) Express as functions of x and y: i) the revenue from selling Jordan shirts, ii) the revenue from selling O'Neal shirts iii) the costs for shirts and iv) the overall profit.

b) Find the critical point of the profit function. Show that it is the unique maximum.