BEEM103

UNIVERSITY OF EXETER

BUSINESS SCHOOL

December 2009
SAMPLE EXAM

OPTIMIZATION TECHNIQUES FOR ECONOMISTS

Module convenor: Dieter Balkenborg

Duration : **TWO HOURS**

The paper has 3 parts. Your marks on the *first part* will be *rounded down* to *55 marks*. Your marks on the *second part* will be *rounded down* to *15 marks.* Obviously, you cannot get over 100 marks overall. This examination is closed book and no materials are allowed. **Full work must be shown** on your script. **Please write legibly.**

Dear Student, this is a *sample exam*, which you receive together with detailed answers to make you familiar with the structure of the January exam. There will also be a *mock exam* to which you are expected to submit your answers in the lecture on January 12th 2009. A further important source of exercises are the class and home exercises. awhich

Part A (You can gain *no more than 55 marks* on this part.)

Problem 1 (10 marks) Simplify

$$\frac{8\sqrt[3]{x^2}\sqrt[4]{y}\sqrt{1/z}}{-2\sqrt[3]{x}\sqrt{y^5}\sqrt{z}} \qquad \left(\left((3a)^{-1}\right)^{-2}\left(2a^{-2}\right)^{-1}\right)/a^{-3}$$

Problem 2 (10 marks) Solve

$$\ln x^{5/2} - 0.5\ln x = \ln 25$$

Problem 3 (10 marks) Consider the function

$$y(x) = e^{-x^2}$$

i) Calculate and draw a sign diagram for the first derivative. Where is the function increasing or decreasing? Are there any peaks or troughs? Does the function have a (global) maximum? Is the function quasi concave?
ii) Calculate and draw a sign diagram for the second derivative. Where is the function convex or concave? Are there any inflection points?

Problem 4 (10 marks) For the function

$$y = \frac{1}{4}x^4 - 2x^2$$

find the (global) maxima and minima a) on the interval $[-1, 1]$ and b) on the interval $[-4, 4]$.

Problem 5 (10 marks) Find the equation of the tangent plane of

$$z(x, y) = \ln(5x + y)$$

at the point $(x^*, y^*, z^*) = (2, 3, z(2, 3))$.

Problem 6 (10 marks) Show that the function

$$u(x, y) = \ln(5x + y) + \ln(x + y)$$

is concave.

Problem 7 (10 marks) Find a solution to the differential equation

$$e^x \frac{dx}{dt} = t$$

Problem 8 (10 marks) Solve the problem

$$\max \int_0^1 \left(1 - x^2 - \dot{x}^2\right) dt,\ x(0) = 0,\ x(1) \geq 0$$

Part B (You can gain *no more than 15 marks* on this part.)

Problem 9 (15 marks) For a consumer with the utility function

$$u(x, y) = -(10 - x)^2 - (16 - 2y)^2$$

derive his demand when a) $p_x = p_y = 25$ and $b = 100$; b) $p_x = 60$, $p_y = 25$ and $b = 100$ and c) $p_x = p_y = 1$ and $b = 100$.

Problem 10 (15 marks) Solve the problem

$$\min \int_0^7 (x(t) - 3)^2 \text{ subject to } x(t) = u(t), \; x(0) = x(7) = 0, \; -1 \leq u(t) \leq 1$$

Part C

Problem 11 (20 marks) Sketch the graph of the area C carved out by the two inequalities

$$\begin{aligned} x^2 + (y-1)^2 &\leq 4 \\ x^2 + (y+1)^2 &\leq 4 \end{aligned}$$

For any point (a, b) in the plane use the Lagrangian approach to determine the point closest to (a, b) within or on the boundary of C. Why can you assume without loss of generality that $a, b \geq 0$? How many cases do we have to consider? Give an argument why you can assume without loss of generality that $a, b \geq 0$.

Problem 12 (20 marks) Two factors, capital, $K(t)$, and an extractive resource, $R(t)$, are used to produce a good, Q, according to the production function $AK^{1-\alpha}R^{a}$ where $0 < \alpha < 1$. The product may be consumed, yielding utility $U(C) = \ln C$, or it may be invested as capital. The total amount of the extractive resource is X_0. Maximize over the finite horizon T utility

$$\int_0^T \ln C(t)\, dt$$

subject to $X' = -R$, $X(0) = X_0$, $X(T) = 0$, $K' = AK^{1-\alpha}R^{\alpha} - C$, $K(0) = K_0$, $C > 0$, $R > 0$. (All parameters are assumed to be positive.)