BEEM103

UNIVERSITY OF EXETER

BUSINESS SCHOOL

December 2009

OPTIMIZATION TECHNIQUES
FOR ECONOMISTS

Mock Exam

Module convenor: Dieter Balkenborg

Duration : **TWO HOURS**

The paper has 3 parts. Your marks on the *first part* will be *rounded down* to *55 marks*. Your marks on the *second part* will be *rounded down* to *15 marks.* Obviously, you cannot get over 100 marks overall. This examination is closed book and no materials are allowed. **Full work must be shown** on your script. **Please write legibly.**

Sumit your answers in the first lecture in December 12th, Room 004, in the Harrison Building.

Part A (You can gain *no more than 55 marks* on this part.)

Problem 1 (5 marks) Simplify

$$\frac{x\sqrt{y}-y\sqrt{x}}{x\sqrt{y}+y\sqrt{x}} \qquad \frac{\sqrt[3]{a}\left((4a)^6\right)^{\frac{1}{12}}\sqrt[4]{a^3}}{(4a^5)^{\frac{1}{12}}\sqrt{a}}$$

Problem 2 (5 marks) Solve

$$\begin{aligned} \ln x^2 - \ln y^2 &= \ln 25 \\ x - y &= 4 \end{aligned}$$

Problem 3 (10 marks) Consider the function

$$y(x) = \frac{1}{1+x^2}$$

i) Calculate and draw a sign diagram for the first derivative. Where is the function increasing or decreasing? Are there any peaks or troughs? Does the function have a (global) maximum. Is the function quasi concave?

ii) Calculate and draw a sign diagram for the second derivative. Where is the function convex or concave. Are there any inflection points?

Problem 4 (10 marks) For the function

$$y(x) = \frac{1}{3}x^3 - x$$

find the (global) maxima and minima a) on the interval $[-2, 0]$ and b) on the interval $[-2, 2]$.

Problem 5 (5 marks) Find the equation of the tangent plane of

$$z(x, y) = x^2 y^2$$

at the point $(x^*, y^*, z^*) = (1, 4, z(1, 4))$.

Problem 6 (15 marks) Show for the Cobb-Douglas production function

$$Q = K^{\frac{1}{3}} L^{\frac{1}{4}}$$

hat the profit function

$$\Pi(K, L) = PK^{\frac{1}{3}} L^{\frac{1}{4}} - rK - wL$$

of price-taking firm facing the output price $P = 10$, the interest rate $r = 5$ and the wage rate $w = 2$ has a unique maximum.

Problem 7 (10 marks) Find a solution to the differential equation

$$\frac{dx}{dt} = t^2 x^2$$

with $x(1) = 1/2$.

Problem 8 (10 marks) Solve the problem

$$\max \int_0^1 \left(10 - \dot{x}^2 - 2x\dot{x} - 5x^2\right) e^{-t} dt,\ x(0) = 1,\ x(1) = 1$$

Part B (You can gain *no more than 15 marks* on this part.)

Problem 9 (15 marks) Consider the area defined by the inequalities

$$\begin{aligned} -1 &\leq x + y \leq 1 \\ -1 &\leq x \leq 1 \end{aligned}$$

Which point in this area is closest to $(5, -5)$?

Problem 10 (15 marks) Find the only possible solution to

$$\max \int \left(x^2 - 2u\right) dt \text{ subject to } \dot{x} = u,\ x(0) = 0,\ x(1) \text{ free}$$

Hint: Show that the co-state variable $q(t)$ is strictly decreasing.

Problem 11 (15 marks) Use the Bordered Hessian to show that the function

$$f(x, y) = e^{-\frac{1}{x^2 + y^2}}$$

is quasiconcave.

Part C

Problem 12 (20 marks) Sketch the graph of the area C carved out by the two inequalities

$$\begin{aligned} x^2 + y^2 &\leq 4 \\ y &\leq x \end{aligned}$$

For any point (a, b) with $a, b \geq 0$ in the plane use the Lagrangian approach to determine the point closest to (a, b) within or on the boundary of C.

(BEEM103 December 2009) (*End of the exam paper.*)