Exercise 1  Solve the problem

$$\max_{u_t \in [0,1]} \sum_{t=0}^{T} \left[ \left( -\frac{2}{3} u_t \right) + \ln x_t \right], \ x_{t+1} = u_t x_t \text{ for } t = 0, \ldots, T - 1, \ x_0 \text{ given}$$

a) by the dynamic programming approach and 
   b) by using the Lagrangian. compare with the Hamiltonian method.

Exercise 2  Consider the problem

$$\max_{u_t \in [-1,1]} \sum_{t=0}^{T} \left( u_t^2 - 2x_t^2 \right)$$

such that $x_{t+1} = u_t, \ x_0 = 0$.

1. Proof that $u_t = 0$ for $t < T$ and $u_T = 1$ or $u_T = -1$ are optimal controls. (Express the objective function as functions of $u_0, \ldots, u_T$ only.

2. Show that $u_t^*$ does not maximize the Hamiltonian.