Exercise 1 Calculate the first and second order partial derivatives of

a) \[ z = (2 - x - y)x + (5 + 2x - 3y^2)y - 3x + 2y^2 \]

b) \[ z = (x^2 + y^3)^5 \]

Exercise 2 Find the critical point of the functions

a) \[ z = f(x, y) = xy - 2x + 3y - 6 \]

b) \[ z = g(x, y) = 2x^2 + 2xy - 6x + 5y^2 - 6y + 5 \]

c) \[ z = h(x, y) = -2x^2 - 2xy + 6x - 5y^2 - 5 \]

Determine whether they are troughs, peaks or saddlepoints.

Exercise 3 A dairy produces whole milk and skim milk in quantities \( x \) and \( y \) gallons, respectively. Suppose that the price of whole milk is \( p(x) = 20 - 5x \) and that the price of skim milk is \( q(y) = 4 - 2y \) and assume that \( C(x, y) = 2xy + 4 \) is the total (!) joint-cost function of the commodities. What should \( x \) and \( y \) be to maximize profit, assuming that the first order conditions yield a maximum? Show that your solution is indeed the unique maximum.