Outline

1. Motivation
2. The Production Function
   • Marginal and Average Products
   • Isoquants
   • The Marginal Rate of Technical Substitution
3. Technological Progress
4. Returns to Scale
5. Some Special Functional Forms

Example: Production of Semiconductor Chips

“fabs” cost $1 to $2 billion to construct and obsolete in 3 to 5 years
⇒ must get fab design “right”
Choice: Robots or Humans?
⇒ Up-front investment in robotics vs. better chip yields and lower labor costs?
⇒ Capital-intensive or labor-intensive production process?

Definitions

Definition: Productive resources, such as labor and capital equipment, that firms use to manufacture goods and services are called inputs or factors of production.

Definition: The amount of goods and services produces by the firm is the firm’s output.

Definition: Production transforms a set of inputs into a set of outputs.

Definition: Technology determines the quantity of output that is feasible to attain for a given set of inputs.

Definitions Continued

Definition: The production function tells us the maximum possible output that can be attained by the firm for any given quantity of inputs.

Example: $Q = f(L,K,M)$

Notes on the Production Function

• Definition: A technically efficient firm is attaining the maximum possible output from its inputs (using whatever technology is appropriate)
Example: The Production Function and Technical Efficiency

Production Function $Q = f(L)$

Production Set

- The variables in the production function are flows (the amount of the input used per unit of time), not stocks (the absolute quantity of the input).

- Example: stock of capital is the total factory installation; flow of capital is the machine hours used per unit of time in production (including depreciation).

- Capital refers to physical capital (definition: goods that are themselves produced goods) and not financial capital (definition: the money required to start or maintain production).

Production Function $Q = K^{1/2}L^{1/2}$ in Table Form

<table>
<thead>
<tr>
<th>K:</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>L:</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
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<td>10</td>
<td>14</td>
<td>17</td>
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<td></td>
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<td>22</td>
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<td>39</td>
<td>45</td>
<td>50</td>
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</tbody>
</table>

Production Function

<table>
<thead>
<tr>
<th>Output from inputs</th>
<th>Preference level from purchases</th>
</tr>
</thead>
<tbody>
<tr>
<td>Derived from technologies</td>
<td>Derived from preferences</td>
</tr>
<tr>
<td>Cardinal (Defn: given amount of inputs yields a unique and specific amount of output)</td>
<td>Ordinal</td>
</tr>
<tr>
<td>Marginal Product</td>
<td>Marginal Utility</td>
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</table>

Utility Function

<table>
<thead>
<tr>
<th>Isoquant (Defn: all possible combinations of inputs that just suffice to produce a given amount of output)</th>
<th>Indifference Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marginal Rate of Technical Substitution</td>
<td>Marginal Rate of Substitution</td>
</tr>
</tbody>
</table>
Definition: The **marginal product** of an input is the change in output that results from a small change in an input holding the levels of all other inputs constant.

\[\text{MP}_L = \frac{\Delta Q}{\Delta L} \text{ (holding constant all other inputs)}\]
\[\text{MP}_K = \frac{\Delta Q}{\Delta K} \text{ (holding constant all other inputs)}\]

Example:
\[\text{MP}_L = \frac{1}{2}L^{-1/2}K^{1/2}\]
\[\text{MP}_K = \frac{1}{2}K^{-1/2}L^{1/2}\]

Definition: The **average product** of an input is equal to the total output that is to be produced divided by the quantity of the input that is used in its production:

\[\text{AP}_L = \frac{Q}{L}\]
\[\text{AP}_K = \frac{Q}{K}\]

Example:
\[\text{AP}_L = \frac{K^{1/2}L^{1/2}}{L} = K^{1/2}L^{-1/2}\]
\[\text{AP}_K = \frac{K^{1/2}L^{1/2}}{K} = L^{1/2}K^{-1/2}\]

Definition: The **law of diminishing marginal returns** states that marginal products (eventually) decline as the quantity used of a single input increases.

**Links between Total, Average and Marginal Magnitudes**

<table>
<thead>
<tr>
<th>Arrival</th>
<th>Height</th>
<th>Total</th>
<th>Average</th>
<th>Marginal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>160</td>
<td>160</td>
<td>160</td>
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<tr>
<td>2</td>
<td>180</td>
<td>340</td>
<td>170</td>
<td>180</td>
</tr>
<tr>
<td>3</td>
<td>190</td>
<td>530</td>
<td>176.67</td>
<td>190</td>
</tr>
<tr>
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<tr>
<td>5</td>
<td>150</td>
<td>830</td>
<td>166</td>
<td>150</td>
</tr>
</tbody>
</table>

"TP*" "AP*" "MP*

• When a total magnitude is rising, the corresponding marginal magnitude is positive.
• When an average magnitude is falling, the corresponding marginal magnitude must be smaller than the average magnitude.

**Isoquants**

Definition: An **isoquant** traces out all the combinations of inputs (labor and capital) that allow that firm to produce the same quantity of output.

Example: \[Q = K^{1/2}L^{1/2}\]

What is the equation of the isoquant for \(Q = 20\)?

\[20 = K^{1/2}L^{1/2}\]

\[\Rightarrow 400 = KL\]

\[\Rightarrow K = 400/L\]
...and the isoquant for $Q = Q^*$?

$Q^* = K^{1/2}L^{1/2}$

$\Rightarrow Q^*^2 = KL$

$\Rightarrow K = Q^*^2/L$
Definition: The **marginal rate of technical substitution** measures the amount of an input, \( L \), the firm would require in exchange for using a little less of another input, \( K \), in order to just be able to produce the same output as before.

\[
MRTS_{L,K} = -\frac{\Delta K}{\Delta L} \quad \text{(for a constant level of output)}
\]

Marginal products and the MRTS are related:

\[
MP_L(\Delta L) + MP_K(\Delta K) = 0
\]

\[
\Rightarrow \quad \frac{MP_L}{MP_K} = \frac{-\Delta K}{\Delta L} = MRTS_{L,K}
\]

therefore,

\[
\text{If both marginal products are positive, the slope of the isoquant is negative...}
\]

\[
\text{If we have diminishing marginal returns, we also have a diminishing marginal rate of technical substitution.}
\]

**For many production functions, marginal products eventually become negative. Why don't most graphs of isoquants include the upwards-sloping portion?**

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**Example:** The Economic and the Uneconomic Regions of Production

![Diagram showing isoquants and marginal products](image)

**Definition:** The **elasticity of substitution**, \( \sigma \), measures how the capital-labor ratio, \( K/L \), changes relative to the change in the MRTS,

\[
\sigma = \left[ \frac{\Delta (K/L)}{\Delta MRTS_{L,K}} \right] \times \frac{MRTS_{L,K}}{(K/L)}
\]

**Example:** Suppose that...

\[
MRTS^A_{L,K} = 4, \quad K^A/L^A = 4
\]

\[
MRTS^B_{L,K} = 1, \quad K^B/L^B = 1
\]

\[
\Delta MRTS_{L,K} = MRTS^B_{L,K} - MRTS^A_{L,K} = -3
\]

\[
\sigma = \left[ \frac{\Delta (K/L)}{\Delta MRTS_{L,K}} \right] \times \frac{MRTS_{L,K}}{(K/L)} = (-3/(-3))(4/4) = 1
\]

---

**Returns To Scale**

How much will output increase when ALL inputs increase by a particular amount?

\[
RTS = \frac{\% \Delta Q}{\% \Delta (\text{all inputs})}
\]

If a 1% increase in all inputs results in a greater than 1% increase in output, then the production function exhibits **increasing returns to scale**.

If a 1% increase in all inputs results in exactly a 1% increase in output, then the production function exhibits **constant returns to scale**.

If a 1% increase in all inputs results in a less than 1% increase in output, then the production function exhibits **decreasing returns to scale**.
Example: Returns to Scale

- If $Q_1 = 2Q$ then constant
- If $Q_1 > 2Q$ then increasing
- If $Q_1 < 2Q$ then decreasing

The marginal product of a single factor may diminish while the returns to scale do not (returns is about changing ALL inputs!!!!!!) whereas marginal return is about changing only one input!!!!!!

Returns to scale need not be the same at different levels of production

Example: $Q_2 = A(L_{1} \alpha K_{1} \beta)$

$Q_2 = A(\lambda L_1)^\alpha(\lambda K_1)^\beta$

$= \lambda^{\alpha+\beta} A L_1^\alpha K_1^\beta$

$= \lambda^{\alpha+\beta} Q_1$

so returns to scale will depend on the value of $\alpha + \beta$.

$\alpha + \beta = 1 \ldots CRS$

$\alpha + \beta < 1 \ldots DRS$

$\alpha + \beta > 1 \ldots IRS$

Special Production Functions

1. Linear Production Function:

$Q = aL + bK$

- $\sigma = \infty$
- MRTS constant
- Constant returns to scale

Example: Linear Production Function

$Q = Q_0$

$Q = Q_1$

2. Fixed Proportions Production Function (Leontief Production Function)

$Q = \min(aL, bK)$

- L-shaped isoquants
- MRTS varies (0, infinity, undefined)

$\sigma = 0$
1. Production function is analogous to utility function and is analyzed by many of the same tools.

2. One of the main differences is that the production function is much easier to infer/measure than the utility function. Both engineering and econometric techniques can be used to do so.

3. **Cobb-Douglas Production Function**: \( Q = aL^\alpha K^\beta \)
   - If \( \alpha + \beta > 1 \) then IRTS
   - If \( \alpha + \beta = 1 \) then CRTS
   - If \( \alpha + \beta < 1 \) then DRTS
   - Smooth isoquants
   - MRTS varies along isoquants
   - \( \sigma = 1 \)

4. **Constant Elasticity of Substitution Production Function**: 
   
   \[
   Q = \left[ aL^\phi + bK^\phi \right]^{1/\phi}
   \]
   
   Where \( \phi = (\sigma-1)/\sigma \)
   
   - If \( \sigma = 0 \), we get Leontief case
   - If \( \sigma = \infty \), we get linear case
   - If \( \sigma = 1 \), we get the Cobb-Douglas case

**Summary**

4. Returns to Scale is a long run concept: It refers to the percentage change in output when all inputs are increased a given percentage.

5. The production function is cardinal, not ordinal