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Abstract

This paper studies corporate governance when a firm operates in imperfect markets. We derive firms’ decisions from utility maximisation by individuals. This reduces the usual monopoly distortion. Corporate governance can effect the equilibrium in the product (or input) markets. This enables us to endogenise the objective function of the firm. If the firm cannot commit not to change its constitution, we find a Coase-like result where all market power is lost in the limit. We present a more abstract model of governance in the presence of market distortions and discuss its implications for the governance of universities.

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1 INTRODUCTION

1.1 Background

A central problem in corporate governance is to explain why firms are organised in the way they are. As Hansmann (1996) shows there are a wide range of firms in reality. These range from the small single owner/manager firm, through large corporations with separated shareholders, bondholders, boards and managers, to worker cooperatives, professional partnerships and hybrid organizations, which include non-profits such as hospitals, charitable organizations, schools and universities. From his discussion of the firm, he characterizes the myriad organizations that have evolved to deal with a wide range of organizational problems. In particular these arise where agents interact strategically in producing complex commodities or services. Factors such as the degree of competition in product and input markets and the presence of asymmetric information have an influence on the nature of the firm.

Hansmann (1996) cites a number of examples where firms are owned either by those who purchase their outputs or those who supply inputs to the firm. He argues that, in most cases, this is to counter monopoly or monopsony power. This practice is very common among firms, which supply inputs to or buy produce from farms. (See also Refsell (1914).) In relatively remote rural areas, it is easier to establish a local monopoly. Likewise lawyers and accountants usually organise as partnerships. The reason for this is similar. The firm is a monopoly supplier of inputs which these people need to work. Partnerships reduce the monopoly distortion.

In the present paper we consider how imperfect competition interacts with the objective function of the firm. For example, consider the labour market where firms hire specialised labour that is industry- and/or firm-specific. Our model allows a firm to take into account the strategic effect of hiring the particular type of labour on the reaction of other firms in the industry. In particular a non-profit firm can pursue a more aggressive strategy in the labour market at the expense of profit maximising rivals. Similar considerations apply if there is
imperfect competition in the product market.

1.2 Modelling Firm’s Decisions

In this paper, we consider an economy with monopoly or oligopoly. As we shall argue, there is a strong case against assuming profit maximisation when markets are distorted. However, it is not clear what the alternative should be. We model the firm as a collection of individuals, each of whom is maximising his/her utility. Decisions are made by a process of aggregating the preferences of a group of agents within the firm.¹

One approach, which has been used in the past, is to model decisions as being made by a majority vote of shareholders.² However one can object to shareholder voting models by arguing that, in practice, management have more influence than shareholders. To model this, we assume that the firm’s decisions are made by a group of individuals, which we shall refer to as the control group. For example, the control group could consist of the shareholders and senior management. As another example, consider a firm with no shareholders, but is a partnership. This case is common in legal, accounting, finance and professional firms, where the firm produces services that are a function of human capital, individual and team effort. However, to preserve generality, we shall not explicitly describe the criteria for membership of the control group.

At present there is no widely accepted theory of the internal structure of the firm³ For this reason we use an abstract model. We make, what we believe to be the mild assumption, that the firm’s procedures respect unanimous preferences within the control group. Such rules would include, inter alia, those which give a major role for management. Note that many familiar forms of governance can be seen as special cases, for instance producer cooperatives,

¹Examples of such procedures would be the Nash bargaining solution used by Hart and Moore (1990), non-cooperative bargaining, de DeMeza and Lockwood (1998), Bolton and Xu (1999) or the voting models used by De Marzo (1993), Kelsey and Milne (1996) and Sadanand and Williamson (1991).
³For recent surveys of the governance literature see Allen and Gale (2000), Becht, Bolton, and Roell (2003), Shleifer and Vishny (1997), and Tirole (2001).
consumer cooperatives, including worker representatives on the board (as in Germany) and many types of non-profit organisation. Despite the generality, our model is able to make a number of predictions concerning equilibrium behaviour and to throw some light on policy questions.

In a discussion of firm structures, Hansmann (1996) provides many examples of firms that are cooperatives, partnerships and non-corporate forms. Some are complex non-profit forms, where the services provided appear to require subtle forms of organisation. Hence it is desirable that any model of the firm should be flexible in abstracting from details that are specific to particular situations and should deal instead with the decision-making process in a general way.

Some theories of the firm use bargaining models to determine the relative power of different individuals. By varying the bargaining game, it is possible to induce different outcomes to the management-control mechanism. Although some of these games have some semblance to reality, we feel they are highly stylized. We prefer to abstract from the details of the bargaining process and simply assume that whatever the bargaining or management game, the process leads to an efficient outcome. If one believes that in certain situations, that the outcome is inefficient, then it would be important to explain the source of the inefficiency. One could think of our model as the outcome of a process to design an efficient mechanism. If this is infeasible then we are dealing with inefficient mechanisms. As this is an open theoretical question, we simply by-pass it by assuming an efficient mechanism exists and explore the consequences of that assumption.

1.3 Corporate Governance and Imperfect Competition

In oligopolistic markets it is often the case that a firm, which aims to maximise profit, will not necessarily make the highest profit. If a given firm deviates from profit maximisation, this can change the behaviour of rivals in ways which give the original firm a strategic advantage. This was demonstrated by Vickers (1985), Fershtman and Judd (1987) and Sklivas (1987),
who argue that owners can increase profits by hiring managers on incentive contracts that reward according to a weighted average of profits and revenues. This makes managers more aggressive, which can raise profits in Cournot oligopoly.

We relate these arguments to the issues of shareholder voting and monopoly. Consider a firm that is the sole producer of a particular good. Assume that there is consumer representation in the control group. We argue that the firm will produce a greater quantity and sell at a lower price than a conventional profit maximising monopolist. A small price reduction will result in a second order loss of profits but a first order gain in their consumer surplus.

In oligopolistic industries there is a similar effect of the firm’s governance on price. In addition, the choice of the firm’s constitution can affect the strategic interaction in markets. Consider a firm in a Cournot oligopoly. Starting at the profit-maximising level, a decrease in price will lower profits but raise consumer surplus. Different individuals will trade-off these effects in different ways depending on their shareholdings and consumption patterns. Suppose a firm gives more weight in its decision procedures to those who have a relatively greater preference for low prices. Then, ceteris paribus, the firm will charge lower prices and produce more output. This will cause rivals to reduce their output thus possibly giving the firm a strategic advantage in the market. Hence increasing influence of consumers on decision-making will, up to a point, increase profits.

The above argument implies that there is an optimal form of corporate governance, which can be derived from considerations of the firm’s position in input or product markets. Consider an entrepreneur who designs the constitution of the firm with a view to selling it to outside investors. Then there is an optimal constitution of the firm, which will maximise its value. This will only be compatible with profit maximisation in exceptional circumstances. We investigate how the optimal constitution varies with the number of firms. The deviation from profit maximisation is greatest when the number of firms is small and tends to zero as it becomes very large.

We have found that the firm can improve its market position by strategically choosing its
constitution. Suppose that the firm does not just choose its constitution once but is able to revise it at any future time. In this case, we obtain a result similar to the Coase conjecture. Consider a firm which is initially profit maximising. The firm will be tempted to change its constitution to increase sales and profits. However the new control group will wish to further amend the constitution to appear more aggressive than it really is. Hence there could potentially be a whole series of expansions of the control group. The result of this process is that the firm will finish by losing all market power and producing the competitive level of output.

Levin and Tadelis (2002) have a related argument. They show that partnerships can be superior to standard firms in the provision of services. This can happen when the quality of the service is not observable. As is well known, a partnership will hire less workers than the corresponding for-profit firm. Where worker ability varies, this results in the partnership hiring higher quality workers and hence producing a better service. Assume that customers cannot observe the quality directly, then they will prefer to purchase from partnerships, which can therefore be more profitable. This has a similar structure to the model in the present paper. In both cases the choice of corporate governance affects the beliefs of other agents. This causes them to change their behaviour, which indirectly affects the profits of the firm. Taking into account these indirect effects a conventional firm may not be the most profitable.

1.4 General Model

The underlying principles behind these results are demonstrated by an abstract model of interaction between firms, which we present in Section 6. In this model, firms play a non-cooperative game where actions are strategic complements or substitutes. In both cases it may be desirable to give influence to an outsider who receives an externality from the firm’s strategic variable.

In the first application we consider Bertrand competition with differentiated goods. We show that, in this case, it may be desirable to have a degree of overlap between the control
groups of two or more firms. To illustrate this point consider two firms competing Bertrand style. Call them firm 1 and firm 2. Suppose that there are shareholders in firm 2 on the control group of firm 1. These individuals will have an incentive to raise the price charged by firm 1 above the profit maximising level, since by doing so they increase the profits of firm 2. However, up to a point, this can help firm 1. Under Bertrand competition there is a strategic advantage in committing to a high price since it encourages other firms to raise their prices. Each firm has an incentive to raise the price charged by the other firm. If both firms do this, it increases joint profits as it moves the industry closer to a collusive equilibrium. Thus firm 1 has an incentive to encourage representatives of rival firms into its control group.

As a second application we consider the implications for the governance of universities. In this case the main influence is externalities rather than imperfect competition. We argue that it is desirable to give alumni influence in the governance of universities. Alumni receive a positive externality from their university since if the university’s reputation increases this raises the value of their own degree and their career prospects. Thus involving alumni in university governance serves to commit the to improve quality.\(^4\)

**Organisation of the Paper** Section 2 explains our model of firm decisions. Its use is illustrated by considering the price and quantity decisions of a uniform pricing monopolist in section 3. The effect of the firm’s objective function on strategic interaction in markets is considered in section 4. The case where the firm is allowed to make multiple revisions to its constitution is modelled in section 5. Section 6 contains the more general model with applications to Bertrand competition and the governance of universities. Section 7 summarises our conclusions. The appendix contains proofs of those results not proved in the text.

\(^4\)See Bowley (2004).
2 FIRMS

2.1 Profit Maximisation

Economists usually assume that firms maximise profits. However the firm’s objective function should be a derived concept. A firm is a collection of individuals, shareholders, managers, workers, customers etc. The firm’s choices come about as a result of maximising behaviour by these individuals. The usual justification for profit maximisation is the Fisher Separation Theorem (see Milne (1974), Milne (1981)), which says that if there are no externalities, the firm has no market power and financial markets are complete, all shareholders will wish to maximise the value of the firm.

In the presence of market distortions, it is not typically the case that owners will wish firms to maximise profits. The Fisher Separation Theorem does not apply if there is imperfect competition, since in that case, a change in the firm’s production plan will affect prices as well as shareholders’ wealth. Firstly, different shareholders will make different trade-offs between more profits and lower prices. Hence, there will be disagreement between different shareholders about the policy of the firm. Secondly, typically, no shareholder will wish to maximise profits. Indeed the concept of profit maximisation is not well defined. Since the firm’s decisions can change relative prices, there is more than one price system which can be used to measure profits. Other market distortions such as incomplete markets\(^5\) or externalities will create similar problems for the objective function of the firm.

As argued above, in the presence of market distortions, shareholder unanimity cannot be guaranteed. Figure 2.1 indicates the problem for a monopolist. The diagram shows the production set for a monopolist who can produce two goods \(X_1\) and \(X_2\). Since the firm has monopoly power, the prices will depend on the firm’s trade. The diagram shows two possible production vectors for the firm. These will give rise to two different price systems. As can be seen from the diagram, individuals \(A\) and \(B\) have opposite preferences over the

\(^5\)Similar issues are discussed in the context of incomplete markets in Kelsey and Milne (1996).
two production plans.

Despite this, it is still the case that there are decisions on which all members of the control group will agree. For instance, we show that a firm, which has a monopoly, will charge less than the profit maximising price. Thus conventional profit-maximising models may have overstated the size of the distortions due to monopoly.

It has been suggested that in addition to shareholders, other parties affected by a firm’s activities should be given influence in the firm’s decisions. These would include *inter alia* representatives of workers, customers and the local community. This paper is able to throw some light on this proposal. Suppose a firm has monopoly power, which cannot be removed by other means. Our model implies that up to a point, increasing customer influence on decisions will reduce distortions. Moreover it could affect competition in the product market. Similarly increasing worker influence can be beneficial if a firm has monopsony power.

2.2 An Alternative Model of the Firm

We model the firm as a collection of individuals, shareholders, managers, workers and possibly customers and other stakeholders. Our aim is to relate the firm’s objective function to optimising behaviour by these individuals. The decisions of firm $f$ are assumed to be made
by a group of individuals $C_f \subset \{1, \ldots, H\}$, which we shall refer to as the control group of firm $f$. The firm’s preferences are assumed to be a function of the preferences of the control group. We do not assume the firm’s preferences are complete or transitive, thus avoiding social choice problems. Note that we do not exclude the possibility that individuals, who are not shareholders (e.g. managers), are able to influence the firm’s preferences. We shall not model the internal decision making of the control group explicitly but simply assume that whatever procedure is used, respects unanimity. Hence, our results do not depend very sensitively on the composition of the control group.

**Assumption 2.1** The firm’s decision procedure respects unanimous preferences of the control group in the sense that if all members of the control group prefer policy $a$ to policy $b$ with at least one strict preference, then the firm will not choose policy $b$.

There is a large literature on the theory of the firm, its objectives and implications for its organization. Some of this assumes a particular objective and explores its implications for product or factor markets when competing with other firms that are profit maximizing.\(^6\) Another related literature tries to derive the firm’s objective as an endogenous implication of a game between players who are either producers or customers of inputs and outputs of the firm. Often the game is described as either a bargaining game or as some non-cooperative game between interested parties to a firm-like organisation. This literature can be characterised as setting up a particular model of the firm that emphasises a particular relationship e.g. human capital acquisition, firm financing or the acquisition of a specific physical asset that gives a player an advantage in acting strategically to determine the actions of the firm.\(^7\)

## 3 MONOPOLY

In this section we study the implications of our model of the firm for monopolies. Recall that a profit-maximising monopolist will price according to the inverse elasticity rule, which

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says that the mark-up of price over marginal cost is inversely proportional to the elasticity of demand.

### 3.1 Price Decisions

Consider a firm which is the sole producer of good $x$. Let $c(x)$ denote the cost of producing quantity $x$. Let $D(p)$ denote the demand when the price of monopoly goods is $p$. The monopolist’s profits are given by $\pi = pD(p) - c(x)$.

**Notation 3.1** We shall assume, without loss of generality, that the control group of the monopolist is $\{h : 1 \leq h \leq M\}$.

Assume that members of the control group have quasi-linear utility functions $u^h = \theta^h \pi + v^h(p)$ for $1 \leq h \leq M$. (Individual $h$ has indirect utility function $v^h$ for consumption of the good produced by the firm.) Since the monopolist implements unanimous preferences of the control group, the optimal point can be obtained by maximising a weighted sum, $\sum_{h=1}^{M} \lambda^h u^h$, of the utilities of the control group for some non-negative weights $\lambda^h$. We may normalise the $\lambda$’s by requiring $\sum_{h=1}^{M} \lambda^h \theta^h = 1$.

A non-profit maximising firm chooses $p$ to maximise: $\sum_{h=1}^{M} \lambda^h u^h = \sum_{h=1}^{M} \lambda^h \theta^h pD(p) - \sum_{h=1}^{M} \lambda^h \theta^h cD(p) - \sum_{h=1}^{M} \lambda^h v^h(p)$. Differentiating with respect to $p$ we obtain, $D(p) + p \frac{dD}{dp} - \frac{dc}{dx} \frac{dD}{dp} + \sum_{h=1}^{M} \lambda^h \frac{dv^h}{dp} = 0$. By Roy’s identity $\frac{dv^h}{dp} = -x^h$, hence, the first order condition may be written as:

$$\frac{(p - \frac{dc}{dx})}{p} = \frac{1}{\eta} \left( 1 - \sum_{h=1}^{M} \lambda^h \frac{x^h}{x} \right),$$  

where $\eta$ is the elasticity of demand and $x^h$ denotes consumption of good $x$ by individual $h$.

As can be seen, the price is given by a modified version of the inverse elasticity rule. If the firm has a single owner-manager, individual $i$, this can be further simplified to

$$\frac{(p - \frac{dc}{dx})}{p} = \frac{1}{\eta} \left( 1 - \frac{x^i}{x} \right).$$  

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8This normalisation is possible provided $\sum_{h=1}^{M} \lambda^h \theta^h \neq 0$. If this were not satisfied, the claimants of the firm’s profit stream would be given no influence over the firm’s decisions. We shall not consider this case further, as we believe it to be of little economic interest.
If the owner consumes all of the firm’s output then the price will be equal to marginal cost, while if (s)he consumes none of the output, this reduces to the usual pricing formula. In general, the optimal price is between marginal cost and the profit maximising level. If the elasticity of demand is constant, then price is lower the greater the owner’s consumption of the monopoly good.

If the control group has multiple members, price is not necessarily equal to marginal cost, even if they consume all of the firm’s output. The price will also depend on the relative bargaining power of different members of the control group. Those with relatively large shareholdings and lower consumption will want higher prices. Other things equal, the price will be lower, the greater the weight given to members of the control group with higher consumption.\(^9\)

### 3.2 Stakeholder Representation

In policy debates on corporate governance it has been argued that firms should not only be run in the interests of shareholders but also other stakeholders. Our model can be used to examine this proposal. We interpret a stakeholder to be an individual who owns no shares but is a worker or consumer. Consider the case where there are two individuals in the control group. Individual 1 is the sole owner. Individual 2 is a “stakeholder” who has no ownership share (hence \(\theta^2 = 0\)) but may nevertheless have influence on decisions.

Our normalisation of the \(\lambda\)’s implies that \(\lambda^1 = 1\), \(0 \leq \lambda^2 < \infty\). Under these assumptions, (1) becomes,

\[
\frac{P_m - \bar{e}_c}{P_m} = \frac{1}{\eta} \left[ 1 - \frac{x^1}{x} - \lambda^2 \frac{x^2}{x} \right].
\]

Increasing the influence of stakeholders would correspond to increasing \(\lambda^2\). By equation (3) this will lower the price of the monopoly good. Hence if competition is impossible, a firm

\(^9\) The problem of a monopolist with some consumers in the control group has been previously considered by Farrell (1985), who assumed unanimity as the firm’s decision rule or Hart and Moore (1996) and Renstrom and Yalcin (2003), who used the median voter rule. Our results are more general since we do not restrict attention to a specific decision procedure.
with some stakeholder representation would be preferable to a profit-maximising monopolist. However, if the power of stakeholders is made too great, price could be reduced below marginal cost, which would be inefficient. In this case, stakeholders would be using their influence to make inefficient transfers from the owners to themselves.

Assume $x_m^1 + x_m^2 = x_m$ i.e. there are no consumers other than the owners and the stakeholders. A social planner would aim to set price equal to marginal cost, i.e. $p_m = \frac{\partial c}{\partial x_m}$. By equation (3) this implies $1 - \frac{x^l}{x} - \lambda^2 \frac{x^l}{x} = 0$, which can be solved to give $\lambda^2 = 1$. Hence the firm should maximise the unweighted sum of utility of shareholders and stakeholders. In practice, this could be implemented by voting over price and giving an equal number of votes to shareholders and stakeholders. In this case, the median voter would choose to set price equal to marginal cost.

### 3.3 Monopsony

Our theory so far has emphasised imperfect competition in the product market and the involvement of consumers in firms’ decisions. However similar reasoning applies if some input markets are imperfectly competitive. This would provide a justification for giving input suppliers a special role in decisions. A common example is where the firm is owned by suppliers of a particular form of labour. It is not uncommon for firms to face imperfect competition in the labour market. The market for labour is often thin. This analysis would also apply if other input markets are imperfectly competitive. Another example is farm marketing cooperatives, which buy the output of farms on imperfectly competitive markets.

We can apply similar reasoning to that used for monopoly. Hence we can reinterpret the first order conditions replacing monopoly with monopsony and demand elasticity with supply elasticity. If there is more than one owner, the price of the input will be between the competitive level and the monopsony level, depending upon the influence of suppliers in the control group. In other words, the monopsony distortion is moderated by the influence of suppliers, and in turn the inefficiency is moderated by including the supplier of the
monopsony input.

When there is more than one monopsony input, the situation is only a little more complicated. Now the quantity of each monopsony input will be determined by the multi-good monopsony pricing rule. Other things equal, the more elastic the supply then the closer the pricing rule approximates the competitive rule and the less importance there is in including the supplier in the control group. In the limit where the supply is perfectly elastic, the supplier plays no effective role in the control decision. Conversely, the more inelastic the supply, the more important the supplier is in affecting the production and input pricing rule.

4 COURNOT OLIGOPOLY

We now consider oligopolistic markets. Most of our analysis of monopoly can be extended to this case. For instance, if those in charge of the firm are, in part, consumers the price will be below the profit maximising level. In addition the constitution of the firm affects strategic interaction in markets. Giving greater representation to individuals who are relatively high consumers of the product is a means to committing to a large output. This is an advantage in Cournot oligopoly. In effect it makes a Cournot oligopolist more like a Stackleberg leader. Hence if there are distortions in the labour and/or product markets, other forms of corporate governance may be superior to conventional profit-maximising firms.

4.1 Model

Consider a Cournot oligopoly with \( n \) firms, which can produce at constant marginal and average cost \( c \). For simplicity we assume a linear inverse demand curve \( p = 1 - \sum_{i=1}^{n} x_i \), where \( x_i \) denotes the output of firm \( i \).

We assume that there are two types of individuals, type \( A \) and type \( B \). Type \( A \) individuals do not consume the industry’s output and \( u^A = \theta^A x^A \).\(^{10}\) Type \( B \) individuals care both about

\(^{10}\)Since there is no uncertainty, there is no serious loss of generality in assuming that the utility of type \( A \) individuals is linear in income.
income and consumption of the output. Consequently they have (indirect) utility functions
\[ u^B = \theta^B \pi^B + v^B (p_x), \]
where \( \pi^t \) denotes total profit income accruing to an individual of type 
\( t \). We consider the case where the firm has a control group which consists of two members.
One of type \( A \) and one of type \( B \). The type \( A \) individual is assumed to own all of the equity.
Thus the utility of the type \( A \) (resp. \( B \)) individual may be written as
\[ u^A = \pi^A = (p_x - c) x_i, \]
(resp. \( u^B = v^B (p_x) \)). The same individual is not represented in the control group of more
than one firm. Our normalisation of the \( \lambda \)'s implies \( \lambda^A = 1, 0 \leq \lambda^B \leq \infty \).

As in the previous section, the decisions of the firm may be represented as maximising
\[ u^A + \lambda_i \lambda^B u^B, \]
after normalisation. We write \( \lambda_i \) for \( \lambda_i^B \). We consider the following 2-stage
game. In the first period, the owners choose \( \lambda_i \) to maximise the value of the firm. In the
second stage, firms compete in quantities Cournot-style. We look for a subgame perfect
equilibrium of the 2-stage game.

**Proposition 4.1** In an \( n \)-firm oligopoly the reaction function of firm \( i \) is given by
\[ x_i = \frac{1 - c + \lambda_i x^B - \left( \sum_{j \neq i} x_j \right)}{2}. \] (4)

**Proof.** Firm \( i \) maximises: \( u^i = u^A + \lambda_i u^B = \left[ 1 - c - \sum_{j=1}^n x_j \right] x_i + \lambda_i u^B \left( 1 - \sum_{j=1}^n x_j \right) \).

The first order condition for optimal choice of \( x_i \) is: \( 1 - c - \sum_{j=1}^n x_j - x_i - \lambda_i u^B = 0 \). By
Roy’s identity \( u^B = -x^B \), hence, \( 1 - c - \sum_{j=1}^n x_j - x_i - \lambda_i x^B = 0 \).\textsuperscript{11} The result follows. \( \blacksquare \)

The higher \( \lambda_i \) the greater the influence given to individual \( B \). The proposition implies
that, ceteris paribus, an increase in \( \lambda_i \) will increase \( x_i \). This makes firm \( i \) more aggressive,
which is advantageous in a game of strategic substitutes. Firms with a larger value of \( \lambda_i \)
will produce higher output in equilibrium. A possible example of this can be found Refsell
(1914), who shows that cooperative grain elevators expanded their output significantly at the
expense of for-profit rivals in the period 1903-1913.

\textsuperscript{11}We assume that the firm takes \( x^B \) (the consumption of good \( x \) by a type-B individual) as given when choos-
ing its output.
**Proposition 4.2** Let $\ell_n$ denote the value of $\lambda_i$ in a symmetric subgame perfect equilibrium with $n$ firms. Then $\ell_n$ is given by

$$\ell_n = \frac{(n - 1)(1 - c)}{(n^2 + 1)x^K}.$$ 

This shows that the optimal value of $\lambda$ tends to 0 as $n$ tends to infinity. The more competitive the market is, the closer firms should stick to profit maximisation. Given that $n$ is restricted to take integer values, the maximum value of $\ell_n$ occurs at $n = 2$ or 3. Thereafter $\ell_n$ is strictly decreasing in $n$. The analysis requires $n \geq 2$ to be economically meaningful. This is intuitive, as $n$ increases the market distortion decreases, thus there is less scope for strategic behaviour. Hence the strategic effect of the firm’s governance is likely to be greatest when the number of firms is small and declines as the market becomes more competitive.

These results generalise. Whenever Cournot oligopoly is a game of strategic substitutes, profit can be raised by giving some influence to consumers. Our results do not depend crucially on assumptions about the preferences of the different individuals. Similar results could be obtained if a firm were owned by a number of individuals who have different preferences between consumption and profits. By adjusting the decision weights of these individuals, the firm can commit to a more or less aggressive policy in the product market. This is demonstrated by the results in section 6, in which a more general form of preferences is used.\(^{12}\)

### 4.2 Optimal Constitution of the Firm

In this section we consider two ways to endogenise the constitution of the firm. The constitution of the firm can be made endogenous by assuming that it is designed by an entrepreneur to maximise the value of the firm. Only in exceptional circumstances would (s)he would choose profit maximisation. Alternatively the constitution of the firm could be designed by a social planner to maximise welfare.

\(^{12}\)Dierker and Grodal (1996) have a result which is almost the reverse of this. They show that under Bertrand competition owners have higher utility if they delegate the running of the firm to a manager with an incentive to maximise profits than if they directly run the firm themselves.
4.2.1 Social Planner

Suppose that the constitution of the firm is chosen by the social planner to maximise the sum of consumer and producer surplus. We assume that for each firm $f$ the $\lambda_f$ is chosen by the social planner. Let $\lambda^*_n$ denote the social planner’s optimal value of $\lambda_f$ in an $n$–firm industry. The social planner is not however able to intervene directly in the markets to make them more competitive or to set prices.

**Proposition 4.3** If the constitutions of the firms are chosen to maximise total surplus $\lambda^*_n = \frac{1-c}{n \pi^*_f}$.

**Proof.** Consider a symmetric equilibrium with $n$ firms. Each firm has $\lambda^f = \lambda^*_n$ and produces output $x^*_n$. From equation (7), $x^*_n = \frac{1}{n} \left( 1 - c - x^*_n + \lambda^*_n x^2 \right)$. Solving,

$$(n + 1) x^*_n = 1 - c + \lambda^*_n x^2.$$

As usual, the social planner will choose to produce where price equals marginal cost, hence, $x^*_n = \frac{1-c}{n}$. Substituting into (5), we obtain $\frac{n+1}{n} (1 - c) = 1 - c + \lambda^*_n x^2$, from which the result follows.

Assume all consumers are represented in the firm, then $x^*_1 = \frac{1-c}{n}$. Substituting into equation (5) we find $\lambda^*_1 = 1$. This implies that shareholders and stakeholders should be given equal influence over the firm’s decisions. Recall we found a similar result for a monopoly. From the social planner’s point of view, the optimal constitution does not depend on the number of firms in the industry.

4.2.2 Entrepreneur

Now suppose that an entrepreneur designs the constitution of the firm to maximise the value at which he can sell it. If the organisation is sold as a profit maximising firm, the price achieved will only be the Cournot oligopoly profits. Higher profits can be made by selling the firm if it has the optimal degree of consumer representation. In this case the entrepreneur will receive the profits of a Stackleberg leader.
Equally if the problem is not one of designing a constitution from scratch, then in a Cournot duopoly, a firm can increase its profits to the Stackleberg level by giving representation to consumers. The market for corporate control may have a similar effect. If the firm did not initially have the optimal form of corporate governance then an outsider could profitably buy up the shares and reorganise the firm. Subsequently the firm could be re-sold at a profit.

4.3 Input Markets

Similar arguments can be used if the firm faces imperfectly competitive input markets. If firms compete in quantities Cournot-style in the labour market, then the firm’s strategic position may be improved by giving workers or their representatives influence in decision-making. We believe that imperfect competition may be more important in input markets than in output markets. This is because labour markets are often highly specialised both by skill and by location. In professions such as law, medicine and education, it is common for some or all suppliers of labour to have more influence than in conventional investor-owned firms. These firms typically require highly specialised labour and face thin markets for this labour. For instance in the UK, there are only 9 paediatric rheumatologists. Clearly with such small numbers competition is not possible. In these circumstances, it may be in the interest of the firm’s owners to give shares to workers or other individuals with an interest in increasing labour demand. (Assuming that these individuals could be prevented from re-selling.)

5 EQUILIBRIUM CONSTITUTION OF THE FIRM

As argued in previous sections, if the founder of a firm wishes to maximise profit it is in his/her interest to choose the constitution of the firm strategically. This is equally true when the other firms do not maximise profit. Whenever the rival firms have a downward sloping reaction function, profit can be increased by adopting a constitution, which commits the

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13Roberts and Steen (2000) have made a similar point. It may be in the interest of a firm to give shares to its workers to encourage investment in firm-specific human capital.
firm to behaving more aggressively. Likewise, the result does not depend crucially on the original objective of the firm. For instance, suppose a consumer cooperative aims to maximise a weighted average of consumer surplus and profits. Such a firm could better achieve its objective by committing to a more aggressive strategy. This would up to a point raise profits and increase consumer surplus because of the strategic effect on other firms’ output. Hence increasing the cooperative’s objective, provided it gives some weight to profits. More generally as long as the current control group gives positive weight to profits, it is in their interest to adopt a constitution which commits the firm to behaving more aggressively than they would choose themselves.

This suggests an alternative way to endogenise the constitution of the firm. We can define the equilibrium constitution of the firm to be such that there is no strategic reason to change the constitution according to the objective of the firm as defined in the constitution itself. This is intended as a theory of the objective function of the firm in a long run equilibrium, in which all possible adjustments have been made.

We obtain a result similar to the Coase conjecture. Consider a firm which is initially profit maximising. The firm will be tempted to change its constitution to increase sales and profits. However if the firm cannot commit to prevent further changes to the constitution, there could be a series of changes each of which increases the firm’s current objective when it was implemented. The result of this process is that the firm will finish by producing the competitive level of output. Unlike the Coase analysis, it is not essential for our argument that the firm’s output be durable.

If, instead of giving influence away, the original owner sold influence then the process may even be in the interest of the original owner. Individuals who consume the firm’s output are always prepared to pay an amount equal to his/her increase in consumer surplus. Up to the Stackleberg point, the owner gets an indirect benefit from selling influence via the strategic effect on profits.
5.1 Model

There are 2 firms, firm 1 and firm 2, which compete Cournot style. For simplicity, we assume a linear inverse demand curve \( p = 1 - x_1 - x_2 \). Firm 2 is a conventional profit-maximising firm. Firm 1 has two members in the control group, one type \( A \) individual and one type of \( B \). Recall type \( A \) (resp. \( B \)) individuals have utilities \( u^A = x^1 \), (resp. \( u^B = v^B (p) \)).

We impose a non-negative profit condition. There are two reasons for this. Firstly as price falls below marginal cost, all other firms, which are assumed to be maximising profit, would exit from the industry. Thus issues of strategic delegation would no longer be relevant. Secondly since the model is intended as one of long run equilibrium. The firm would not be viable in the long run if it makes losses. The non-negative profit condition can also be justified since limited liability implies that owners cannot be forced to contribute additional funds to the enterprise.

**Assumption 5.1** Firms cannot make losses.

This implies that, price must be greater than or equal to marginal cost, \( p \geq c \). The following result demonstrates that, in an equilibrium in which the firm does not wish to change its constitution, price will equal marginal cost.

**Proposition 5.1** Under Assumption 5.1, the only equilibrium constitution is where \( \lambda^B = \lambda^{**B} = \frac{1-c}{p^2} \), \( x_1 = 1 - c \) and \( p = c \).

This implies that, in the absence of commitment, the firm will increasingly delegate more power to consumers’ representatives.

5.2 Discussion

We have argued that by a process of successive strategic delegation, a firm can become taken-over by its customers. At first sight this may appear implausible. However we believe this story does capture some aspects of reality. Firstly it should be noted that a customer may be
another firm. There are documented cases in which upstream firms have been taken over by downstream firms, including the well-known takeover of Fisher Body by General Motors. Another example is the purchase by farmers of firms which supply inputs (e.g. fertilizer) to farms, see Hansmann (1996) and Refsell (1914).

An analogous story could be told in terms of imperfectly competitive input markets. The conclusion would be that successive rounds of delegation would hand control to suppliers, who would bid more aggressively in the input market. In this case we would see the suppliers of inputs would eventually take over the firm. If the input is top-level management, there is evidence that such a takeover has indeed happened, see Roe (1994).

An alternative way to interpret this result is that firms will have incentives to adopt devices, which preclude too much strategic delegation to prevent loss of control. Firms do indeed adopt different procedures for different kinds of decisions. Pricing and output decisions are usually made by managers, while mergers and takeovers require the approval of shareholders.

The initial controlling group has an interest to commit to no further strategic delegation after the first stage. If such commitment is not possible, then a far-sighted owner may not permit the first round of strategic delegation foreseeing that it will trigger a whole series of successive delegations, which will ultimately have the effect of reducing his/her profit.14

6 EXTENSIONS

In previous sections we have shown that giving influence to consumer representatives can be a good strategy under Cournot competition. Here we show this can be generalised to a more

14A related result can be found in Baye, Crocker, and Ju (1996). They show that firms in Cournot oligopoly have an incentive to divide themselves into competing divisions. The benefit of divisionalisation is that it has a strategic effect on the output of rivals. As the cost of creating new divisions tends to zero, price converges to marginal cost. Again lack of commitment can lead to excessive divisionalisation and a complete loss of market power.
abstract model of competition, where firms’ actions may be either strategic complements or substitutes. In both cases it may be desirable to give influence to an outsider who receives an externality from the firm’s strategic variable. As an application we show that under Bertrand competition is it desirable to give some influence to representatives of competitor firms. This provides a possible rationale for systems of cross shareholdings and directorships seen in certain industries and in certain countries. In a second application we consider the implications for the governance of universities.

6.1 A More General Model

There are $n$ firms. Firm $i$ chooses a strategic variable $x_i$ from its strategy set $X_i$, which we assume to be a closed interval in $\mathbb{R}$. The profits of firm 1 are given by $\tilde{\pi}^1(x_1, x_{-1}) = \pi^1(x_1, \phi(x_{-1}))$, where $\phi : X_{-1} \to \mathbb{R}$, is increasing. Thus the profits of firm 1 depend on its own action and a 1-dimensional aggregate of the actions of its rivals. We assume that $\pi^{1}_{11} < 0$, hence firm 1’s profit is a concave function of its own strategy. Let $R(x_1)$ denote the best response of firm 1’s rivals, which is assumed to be unique.\footnote{This could be justified by assuming that the rivals’ objective functions are strictly concave in their own strategies.}

The control group of firm 1 potentially consists of $m + 1$ individuals, $0 \leq i \leq m$. Individual 0 is only concerned about the profits he receives from firm 0. He/she has utility function $u^0(\pi^0)$. For $1 \leq i \leq n$, individual $i$ has utility function $u^i(\pi^1, x^1)$. We assume that $u^i_{22} < 0$. As in previous sections, firm 0 may be represented as maximising, $\sum_{i=0}^{m} \lambda^i u^i$, for some weights $\lambda^i$.

We consider the following 2-stage game. First firm 1 chooses the $\lambda^i$’s. In the second stage, all firms choose their strategic variables simultaneously and independently.

**Proposition 6.1** An optimal value of the $\lambda$’s is to set $\lambda^i = 0$, $i \neq 0$, $j$;

$$
\lambda^0 = 1 - \frac{\pi^1_2 \phi' R' (x_1)}{\frac{\partial u^0}{\partial x_1}} \frac{\frac{\partial u^0}{\partial x^i}}{\frac{\partial u^0}{\partial x^j}}, \lambda^j = \frac{\pi^1_2 \phi' R' (x_1)}{\frac{\partial u^0}{\partial x^j}}.
$$

15This could be justified by assuming that the rivals’ objective functions are strictly concave in their own strategies.
Note that this result does not assume that rival firms are maximising profit. Nor indeed is it necessary that firm 1’s objective is profit maximisation. Incentive compatibility implies that it will be not be possible to implement a negative value of \( \lambda^j \). However if some individuals receive positive externalities from the firm’s strategic variable, while others receive negative externalities there will always be an appropriate kind of outsider who can be given influence.

As an example consider a Cournot duopoly where firms’ quantities are strategic substitutes, hence \( R' < 0 \). In this case an increase in output by one firm reduces profits at all of its rivals hence \( \pi_2^1 < 0 \). Thus provided individual \( j \) gets a positive externality (i.e. \( \frac{\partial \mu^j}{\partial x_1} > 0 \)) from firm 1’s strategic variable, the optimum can be implemented with a positive value of \( \lambda^j \). A consumer is likely to receive a positive externality if a firm in an imperfectly competitive industry expands output. Thus this confirms our earlier result about the desirability of consumer representation in Cournot oligopoly.

If \( m > 1 \), the optimal values of the \( \lambda \)’s are not unique, since the first order conditions are linearly dependent. This result shows that the optimum may be achieved by a relatively simple constitution for the firm. The control group need consist of only two individuals, one of which is only interested in profit. The other has either positive or negative externalities (as appropriate) from the firm’s strategic variable.

6.2 Price Competition

6.2.1 Model

This section is concerned with what happens when the control groups of two firms overlap. Hence the same individual has influence over the decisions of two or more firms. We show that if firms produce differentiated goods and compete Bertrand style, it is desirable to have representatives from rival firms in the control group. If firms compete in prices there is a strategic advantage to committing to higher prices. Overlapping control groups provides one way to do this.

If firms compete in prices, the strategic variables will usually be strategic complements.
In this case \( R' > 0 \) and \( \pi_2^1 > 0 \). Thus equation (6) implies that it is desirable to share influence with an individual who gets a positive externality from the firm’s strategic variable, \( \frac{\partial \pi_i}{\partial \pi_j} > 0 \). To illustrate this consider two firms, \( i = 1, 2 \), which produce differentiated goods at zero average and marginal cost. We assume that firm \( i \) faces a linear demand curve: 
\[
D_i(p_i, p_j) = \max\{0, a + p_j - c p_i\}, \quad a, c > 0, c > 1,
\]
where \( p_i \) denotes the price of firm \( i \). There are two types of individuals, type \( A \) and type \( C \). Both types do not consume the industry output but can own shares in the firms. In particular \( u^A = \pi_1 \) and \( u^C = \pi_2 \). As before we may represent the objective of firm 1 (resp. 2) as maximising \( u^A + \lambda_1 u^C \) (resp. \( u^C + \lambda_2 u^A \)). The interpretation is that by including a type \( C \) individual in the control group of firm 1, we are giving influence to an individual who is, in some sense, a representative of firm 2.

Firms play a two-stage game, where in the first stage they choose the degree of influence given to a representative of the other firm, \( \lambda_i \), and in the second stage the firms compete Bertrand-style in prices. The following result finds the price in a sub-game perfect equilibrium where both firms use the same \( \lambda \).

**Proposition 6.2** In a symmetric equilibrium where both firms choose the degree of outsider representation, the value of \( \lambda \) is given by:

\[
\lambda = \frac{1}{2c - 1}.
\]

As \( c \) increases products become more differentiated and hence the equilibrium \( \lambda \) decreases. The less products are differentiated, the more representation is given to owners of the other firm. When products are less differentiated competition is more intense. This increases the desirability of softening competition by giving representation to the owners of the other firm. If \( c = 1 \), the market is a Bertrand oligopoly with homogenous goods. In this case the optimal \( \lambda \) is 1. This implies that both firms maximise the sum of their profits. The limiting case where \( c \to \infty \) corresponds to two independent monopolists operating on unrelated demand curves. In this case the optimal \( \lambda \) is 0, since when there is no relation

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between the demand curves, there is no advantage to involving owners of the other firm in the control group.

6.2.2 Discussion

There are a number of examples of situations where the same economic agent may have influence in two or more firms pursuing related lines of business. Visa and Mastercard have an effective duopoly over credit cards. They are controlled by the member banks. Many banks are members of both systems hence have some influence over the running of both credit card systems. Links between control groups are very common in the car and airline industries. Industrial groups, as seen in Japan and Korea, may also be in part motivated by the effects discussed in this section. It is clear from recent empirical work that interlocking ownership plays a major role in oligopolistic industries in many countries. For instance Mork, Stangeland, and Yeung (2000) show that pyramidal control structures are common in a number of countries such as Canada, Japan and Germany. Even in the USA, overlapping directorships are not unknown.

The practices described in this section are clearly collusive. Public policy should try to discourage them. In this paper we assume that there is no regulation. However our results make a good case why regulation is desirable.

6.3 Universities

As a second example, we argue our framework can help to understand the governance of universities, especially private universities. Many universities involve alumni, especially prominent donors in their decision making.\footnote{In Oxford and Cambridge all MA’s have voting rights on a number of key issues. (Including the election of the Oxford Professor of poetry.) Since all graduates are entitled to an MA five years after graduating there is very widespread alumni involvement.} We argue that, under some reasonable assumptions, this is a prediction of our theory.

Assume that increasing the quality of one university will increase both the marginal and
total benefit of raising quality in other universities. (It seems reasonable to assume that these effects are stronger at universities which are geographically close.) This means both that there are positive externalities between neighbouring universities and there is strategic complementarity in the sense that improving the quality of one university will increase the incentives to improve quality at nearby universities.\footnote{It can reasonably be objected that the quality of a university is not adequately measured by a single real number. However we note that multi-dimensional theories of strategic complementarity exist (see Milgrom and Roberts (1990), Milgrom and Shannon (1994)) hence we remain confident that our results would still hold in a more realistic setting where we allow for multiple aspects of quality.} One reason for this is that there is a positive effect of competition. An improvement in the quality of a nearby university sets a higher benchmark for performance at the home institution. Another might be that increases in quality at other universities improves the pool from which research collaborators and new faculty can be drawn.\footnote{There may also be some more direct externalities if there is cross teaching between nearby universities especially joint PhD programmes.} Evidence for strategic complementarity between neighbouring universities is provided by the fact that universities tend to concentrated in certain regions, Paris, the London area, and the Boston area being leading examples.

Consider equation (6), interpret the firms as universities and $x_1$ to be the quality of university $i$. If there are positive externalities between different universities then $\pi_1 > 0$, strategic complementarity implies $R' > 0$. Equation (6) implies it is desirable to give influence in university governance to an individual for which $\frac{\partial w_i}{\partial x_1} > 0$, i.e. an individual who gets positive externalities from the quality of the university. We shall argue that the common practice of involving alumni in university governance is an example.

Alumni receive positive externalities from the quality of their own universities, since improvements in quality will increase the prestige of their own degrees and hence their career prospects. Thus alumni involvement serves as commitment device to increase the quality of the university. Alumni are used rather than current students since the quality of programmes may only become apparent after a period of time. In addition alumni have greater incentives...
to maintain standards than current students. Thus when there are positive externalities and strategic complementarity, increasing alumni involvement increases the quality of a given university and all other universities. The outcome will be Pareto superior compared to the Nash equilibrium where both firms are maximising profit. If there are multiple equilibria, high alumni involvement can act as a selection device moving the university system to a higher level equilibrium.

There is a second advantage of alumni involvement in university governance. Assume that potential students may not observe university quality directly. Instead the quality of education only gradually becomes apparent after graduation through its effect on the quality of life and career prospects. Alumni involvement may act as a commitment device to maintain quality. Potential students can observe the organisation of the university and hence would be attracted to a university with a high degree of alumni involvement.

A more complex model would take into account the fact that universities are engaged in both teaching and research. Both faculty and alumni are actively engaged in the running of universities. This may be explained by postulating that faculty get positive externalities from the quality of research and that there is strategic complementarity between teaching and research.

Our model of a university should be seen as a generalisation of the Glaeser and Shleifer (2001), and Glaeser (2003) model of a non-profit organization. In Glaeser (2003) there are many examples of non-profit firms explored e.g. hospitals, art museums and universities. Their model has a firm that produces output and a quality attribute. The firm is controlled by a manager who maximises utility, but can be influenced by the actions of workers. Clearly this model can be seen as a version of our model where university reputation is the quality attribute and the manager and worker (academic faculty) influence is just a special case of our control group. We have stressed the role of the alumni acting in the control group, but it is easy to extend the model to include other groups. An additional advantage of our model is that we allow more general market structures - the case of a competitive market with for-
profit competitors is just a special case.

7 CONCLUSION

In this paper we have argued that a number of apparently unrelated issues in the governance of organisations can be explained as responses to market distortions. These include but are not limited to the use of professional partnerships, farm marketing cooperatives and the role of alumni in university governance. In particular, in the presence of imperfect competition appropriate choice of corporate governance can reduce market distortions and/or shift the equilibrium in the favour of a given firm. Moreover the general model shows that similar considerations apply to other market distortions. Our conclusions are supported by the evidence in Hansmann (1996).

According to our arguments when there is monopoly or Cournot-style competition there are incentives for increased consumer involvement in the governance of firms. Clearly this does not happen in all instances of monopoly or imperfect competition. We conjecture that is because the costs to consumers of organising varies between different firms. An important factor, which affects the costs of organisation is geographical distance. If customers live close together they can meet and organise more cheaply. Thus the customers of a grain elevator can organise relatively cheaply since they are relatively small in numbers and live close together compared to the customers of Microsoft.

A second, possibly more important, cost of organising is the cost of collective decisions. All systems of collective choice impose costs, both direct costs of operating the mechanism and indirect costs if the outcome of the mechanism is inefficient. Costs of collective decisions are greater the more diverse the preferences of the group of individuals making the decision. This provides a second reason why the customers of a grain elevator are able to organise, while the customers of Microsoft are not. Microsoft customers are much more diverse which means that they are more likely to suffer from the various problems of voting such as the well known Condorcet paradox. For further discussion see Hansmann (1996) pp. 39-44.
Our arguments imply that, in general, consumer representation increases profits in Cournot competition and representation from rival firms increases profits under Bertrand competition. This suggests that we would expect to see different patterns of organisation of firms under the two types of competition. This hypothesis can, in principle, be tested.

We believe an advantage of our model is that it provides an endogenous theory of corporate governance. This can be done in three ways, the firm’s constitution could be chosen by an entrepreneur to maximise the value at which the firm can be sold; the firm’s constitution could be chosen by those currently controlling the firm to maximise their objectives or the system of corporate governance could be chosen by a social planner to maximise social welfare. Although a number of suggestions have been made, at present economics lacks a well established theory of corporate governance. All three proposals have validity beyond the present context.

More generally we believe that there needs to be a rethinking of many results from industrial organisation to allow for more detailed modelling of the internal organisation of the firm. As an example consider management buy-outs (MBO’s). Much of the existing literature has used an agency theoretic approach. It is argued that their main benefit is improved incentives for management. In the present paper we argue that the changes in corporate governance can affect a firm’s position in the product and/or labour markets. The main effect of an MBO is to transfer control of the firm from investors to suppliers of managerial labour. If the managerial labour market is imperfectly competitive this could have the effect of improving the firm’s strategic position in that market.

A possible direction for the future is that skilled labour will become more important. This would shift the bargaining power within organisations. In the firm of the future it is possible that capital will be hired by a coalition of skilled workers. The model in the present paper may help us to understand such changes.

APPENDIX
A  Oligopoly

This appendix contains proofs of our results on oligopoly.

A.1 Cournot Model

Proof of Proposition 4.2  We shall look for a symmetric equilibrium where all firms use the same value of $\lambda = \ell_n$ and produce the same output, $\hat{x}$. Using equation (4) we find,

$$\hat{x} = \frac{1 - c - (n - 1)\lambda + \ell_n x^B}{2}.$$  

Solving, $\frac{n + 1}{n + 1} \hat{x} = \frac{1 - c + \ell_n x^B}{2}$. Hence equilibrium output is given by,

$$\hat{x} = \frac{1 - c + \ell_n x^B}{n + 1}.$$  

We shall now look for the Stackelberg equilibrium, in which firm $i$ is the leader and the outputs of the other firms lie on the equilibrium reaction function:

$$x_{-i} = \frac{n - 1}{n} \left(1 - c - x_i + \ell_n x^B\right),$$

where $x_{-i} = \sum_{j \neq i} x_j$ denotes the total output of all firms other than $i$. Firm $i$’s profit is given by, $\pi_i = (1 - c - x_{-i} - x_i) x_i$. Substituting from the reaction function, $\pi_i = \left(\frac{1}{n} (1 - c) - \frac{n - 1}{n} \ell_n x^B - \frac{1}{n} x_i\right) x_i$. The first order condition is, $(1 - c) - (n - 1) \ell_n x^B - 2x_i = 0$. Hence firm $i$’s Stackelberg output is, $x_i = \frac{(1 - c) - (n - 1) \ell_n x^B}{2}$.  

The level of $\lambda_i$, which maximises firm $i$’s profit, is achieved where the equilibrium output is equal to the Stackelberg output. Hence $\frac{1 - c + \ell_n x^B}{n + 1} = \frac{(1 - c) - (n - 1) \ell_n x^B}{2}$. Solving for $\ell_n$:

$$2(1 - c) + 2\ell_n x^B = (1 - c)(n + 1) - (n^2 - 1) \ell_n x^B \quad \text{or} \quad (n^2 + 1) \ell_n x^B = (1 - c)(n - 1),$$

from which the result follows.  

Lemma A.1  In a Cournot duopoly where firm 2 maximises profit, the market equilibrium is given by,

$$\bar{x}_1 = \frac{1 - c + 2\lambda x^B}{3}, \quad \bar{x}_2 = \frac{1 - c - \lambda x^B}{3}, \quad \bar{p} = \frac{1 + 2c - \lambda x^B}{3}.$$  

Proof.  By Proposition 4.1, the reaction function of firm 1 (resp. 2) is given by $x_1 = \frac{(1 - c - x_2) + \lambda x^B}{2}$, (resp. $x_2 = \frac{(1 - c - x_1)}{2}$). Solving in the usual way we obtain, $\bar{x}_1 = \frac{1 - c + 2\lambda x^B}{3}$.
and \( \bar{x}_2 = \frac{1-c-j^B x_B}{3} \). The equilibrium price is given by, \( \bar{p} = 1 - \frac{1-c+j^B x_B}{3} - \frac{1-c-j^B x_B}{3} = \frac{1+2c-j^B x_B}{3} \).

**Proof of Proposition 5.1** To check \( \lambda^B = \lambda^{*B} \) is indeed an equilibrium. If \( \lambda^B = \bar{\lambda}^B \) then from (8) the equilibrium quantity and price will be \( x_1 = 1 - c \) and \( p = c \).

As before, we may represent the firm’s objective as: \( \psi_1 = \pi_1 + \bar{\lambda}^B u^B = \frac{1}{9} (1-c)^2 + \frac{1}{9} \left( \left( 1-c \right) \lambda^B x_B - 2\lambda^B x_B x_B^2 \right) + \frac{1}{3} \lambda^B B \left( \frac{1+2c-j^B x_B}{3} \right) \) for \( \lambda^B, \bar{\lambda}^B \leq \frac{1-c}{x^B} \). From the point of view of the equilibrium constitution, the effect of a change in \( \lambda^B \) on the firm’s objective is given by \( \frac{d\psi_1}{dx} = \frac{1}{9} \left( (1-c) x_B - 4 \lambda^B x_B^2 \right) + \frac{1}{9} x_B \lambda^B = \frac{1}{9} (1-c) x_B - 4 (1-c) x_B + \frac{1}{9} (1-c) x_B = 0 \), which establishes that \( \lambda^B = \lambda^{*B} \), is an optimum and hence that the firm would not want to decrease \( \lambda^B \) below \( \lambda^{*B} \).

The firm has no incentive to increase \( \lambda^B \), since it is already supplying the entire market. Firm 2’s output is zero, hence there is no strategic effect of further reductions in \( \lambda^B \). The only effect of reducing \( \lambda^B \) further would be to cause the firm to increase its output beyond the current level. This is not desirable as the current level is already optimal according to the current objective function.

**Uniqueness.** Assume \( \bar{\lambda}^B < \frac{1-c}{x^B} \). Then \( \frac{d\psi_1}{dx} \bigg|_{\lambda^B = \lambda^{*B}} = \frac{1}{9} \left( (1-c) x_B - 4 \lambda^B x_B^2 \right) + \frac{1}{9} x_B \lambda^B = \frac{x_B}{9} \left( (1-c) + \frac{\bar{\lambda}^B}{3} \right) (3-4x^B) \).

If we assume \( x^B \leq \frac{3}{4} \) then it is clear that \( \frac{d\psi_1}{dx} \bigg|_{\lambda^B = \lambda^{*B}} > 0 \). If \( x^B > \frac{3}{4} \), \( \frac{d\psi_1}{dx} \bigg|_{\lambda^B = \lambda^{*B}} > \frac{x_B}{9} \left( (1-c) + \frac{1-c}{x^B} (3-4x^B) \right) = \frac{(1-c)}{3} (1-x^B) > 0 \), since \( x^B < 1 \). This implies that if \( \bar{\lambda}^B < \frac{1-c}{x^B} \) there is a strategic advantage to increasing \( \lambda^B \). Hence \( \lambda^B < \frac{1-c}{x^B} \) is not compatible with equilibrium.

If \( \lambda^B > \frac{1-c}{x^B} \), price would be below marginal cost and hence would not satisfy the non-negative profit constraint. Consequently such values of \( \lambda^B \) would not be sustainable in long run equilibrium. ■

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19 This is the point at which the rival exits the market, which causes the profit function to be non-differentiable.
A.2 General Model

Proposition 6.1 An optimal value of the $\lambda$’s is to set $\lambda^i = 0$, $i \neq 0$, $j$;

$$\lambda^0 = 1 - \frac{\frac{1}{2} \phi' R'(x_1) \frac{\partial u^i}{\partial x_1}}{\frac{\partial u^0}{\partial x_1}} , \lambda^j = \frac{\frac{1}{2} \phi' R'(x_1)}{\frac{\partial u^i}{\partial x_1}} .$$

Proof of Proposition 6.1 If firm 1 can act as a Stackelberg leader, its profit will be given by, $\pi^1(x_1, \phi(R(x_1)))$. The first order condition for maximising this is:

$$\pi^1_i + \pi^1 \frac{1}{2} \phi' (R(x_1)) R'(x_1) = 0 .$$

Firm 1’s actual first order condition is, $\sum_{i=0}^{m} \lambda^j \frac{\partial u^i}{\partial x_1} \pi^1_i + \lambda^j \frac{\partial u^j}{\partial x_1} = 0$. Assume that $\lambda^i = 0$, unless $i = 0$ or $i = j$. This simplifies to $\lambda^0 \frac{\partial u^0}{\partial x_1} \pi^1_i + \lambda^j \frac{\partial u^j}{\partial x_1} = 0$. The $\lambda$’s are only unique up to positive scalar multiple. Hence we may normalise them by requiring $\lambda^0 \frac{\partial u^0}{\partial x_1} + \lambda^j \frac{\partial u^j}{\partial x_1} = 1$. This simplifies the first order condition to

$$\pi^1_i + \lambda^j \frac{\partial u^j}{\partial x_1} = 0 .$$

Comparing (10) and (11) we see that if

$$\lambda^j \frac{\partial u^j}{\partial x_1} = \frac{1}{2} \phi' R'(x_1) ,$$

the firm can obtain profits as if it were a Stackelberg leader. Since this sets an upper bound to the profits firm 1 can make in the second stage, it follows that this is the optimal value for $\lambda^j$. Solving for $\lambda^0$, we obtain $\lambda^0 = 1 - \lambda^j \frac{\partial u^j}{\partial x_1} = 1 - \frac{1}{2} \phi' R(x_1) \frac{\partial u^j}{\partial x_1}$.$\blacksquare$

The following results apply to the model of price competition from Section 6.2.

Lemma A.2 In a symmetric Bertrand equilibrium where $\lambda_2^A = \lambda_1^C = \lambda$, prices are given by

$$\hat{p} = \frac{a}{2c(a + p_2)} .$$

Proof. Firm 1’s objective is to maximise: $u^A + \lambda_1^C u^C = p_1 (a + p_2 - cp_1) + \lambda_1^C p_2 (a + p_1 - cp_2)$.

The first order condition is: $a + p_2 - 2cp_1 + \lambda_1^C b p_2 = 0$. Thus firm 1’s reaction curve is given by,

$$p_1 = \frac{a + (\lambda_1^C + 1) p_2}{2c} .$$

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Let \( \hat{p} \) denote the symmetric equilibrium price, where both firms use the same \( \lambda \). This satisfies
\[
\left[ \frac{2c\lambda}{2c} \right] \hat{p} = \frac{a}{2c^2},
\]
from which the result follows. ■

Lemma A.3 Assume that firm 1 maximises profit, while firm 2 gives weight \( \lambda^A \) to type A individuals in its decisions. Suppose that firm 1 is a price leader, then its optimal price is given by:
\[
\hat{p}_1 = \frac{a(2c + 1)}{2c^2 - (\lambda^A + 1)}
\]

Proof. From equation (13) and symmetry, firm 2’s reaction function is given by: \( p_2 = \frac{a}{2c} + \frac{(\lambda^A + 1)p_1}{2c} \). Firm 1’s profits are given by: \( \pi_1 = p_1 \left( a + \frac{a}{2c} + \frac{(\lambda^A + 1)p_1}{2c} - cp_1 \right) \). The first order condition for profit maximisation is:
\[
\frac{d\pi_1}{dp_1} = \left( a + \frac{a}{2c} + \frac{(\lambda^A + 1)p_1}{c} - 2cp_1 \right) = 0.
\]
Hence \( 2ca + a + 2 \left( \lambda^A + 1 - 2c^2 \right) p_1 = 0 \), from which the result follows.\(^{20} \)

Proposition 6.2 In a symmetric equilibrium where both firms choose the degree of outsider representation, the value of \( \lambda \) is given by: \( \lambda = \frac{1}{2c-1} \).

Proof of Proposition 6.2 At the optimum \( \lambda \) the price leader’s price is equal to the equilibrium price:
\[
\hat{p}_1 = \frac{a(2c+1)}{2c^2 - (\lambda + 1)} = \frac{a}{2c}.
\]
Cross-multiplying: \( (2c + 1)(2c - (\lambda + 1)) = 2(2c^2 - (\lambda + 1)) \). Expanding, \( 4c^2 + 2c - 2c\lambda - 2c - (\lambda + 1) = 4c^2 - 2(\lambda + 1) \), hence \( 2c\lambda = \lambda + 1 \) or \( \lambda = \frac{1}{2c-1} \), from which the result follows. ■

References


\(^{20}\)From (14), \( \frac{d^2\pi_1}{dp_1^2} = \frac{1}{c} \cdot \left[ \frac{\lambda^A}{2c^2} + 1 - 2c^2 \right] \leq 0 \), provided \( \lambda^A \leq 2c^2 \). Since \( c \geq 1 \), the second order condition is satisfied provided \( \lambda^A \leq 1 \). This says that type-A individuals do not get more weight than type-C individuals in firm 2’s decisions. In other words outsiders get less than half of the control rights. It does not seem unreasonable to view this as the normal case.


