A six-component contact force measurement device based on the Stewart platform

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Abstract: This paper describes the design of a six-component contact force measuring device, the purpose of which is to verify a system of theories that predict the forces found in robotic grasping. The geometry of the device is based upon that of the Stewart platform manipulator, configured symmetrically. The elastic properties of the device as a whole, and of its legs, are analysed and chosen so that the device possesses appropriate characteristics. The resulting design is described, and its calibration and use are reported. Particular attention is paid to the synthesis of the elastic properties of the system, including an elastic central strut to maintain tension in the legs, which themselves are designed for acceptable sensitivity. The whole device is shown to give good results in contact force measurements.

Keywords: contact force measurement, Stewart platform, screw algebra, stiffness, force and torque transducers, robotics, grasping, kinematics, statics, design

NOTATION

Physical scalar quantities are given in lower case lettering in italic type, vectors in lower case lettering in bold italic type and matrices in upper case lettering in bold upright type.

- \( a \) pitch circle radius of the base leg mounts of the device
- \( b \) pitch circle radius of the platform leg mounts of the device
- \( C_A \) calibrated augmented matrix
- \( E \) modulus of elasticity of the cantilevers
- \( E_s \) \( 6 \times n \) matrix aggregating the strain gauge output from the six legs as columns in calibration trials
- \( f \) six-element vector of the magnitudes of forces in the six legs
- \( f_e \) three-element vector of an external force
- \( f_n \) three-element vector of the force along a contact normal
- \( f_p \) six-element vector of the magnitudes of forces in the legs, resulting from the preload
- \( f_t \) three-element vector of the force acting on the tangent of a contact
- \( f_i \) magnitude of the tensile force in the \( i \)th leg
- \( f_o \) magnitude of the force in the central strut
- \( f_{po} \) magnitude of the preload force in the central strut
- \( h \) equivalent platform height
- \( J \) matrix whose columns are the unit screws of zero pitch, representing the axes of the legs
- \( J_A \) augmented matrix incorporating the central strut
- \( k_l \) axial stictiss of a leg
- \( k_o \) axial stictiss of the central strut
- \( K \) global stiffness matrix incorporating the legs alone
- \( K_A \) global augmented stiffness matrix including the effect from the central strut
- \( m_e \) moment of an external force with respect to the global origin
- \( p_h \) pitch of a screw
- \( r \) radius of the hemispherical tip
- \( r \) position vector of the contact point
- \( r_o \) perpendicular vector between the position vector and the global origin
- \( T \) infinitesimal twist of the platform
- \( u_e \) axial elastic extension of the central strut owing to an external load
- \( u_{po} \) axial elastic extension of the central strut owing to a preload
- \( u \) vector of the axial elastic extension of the six legs
- \( u_p \) vector of the axial elastic extension of the six legs owing to a preload

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The external wrench applied to the hemispherical platform

\( w_e \) nominal external wrench applied to the device

\( W \) matrix of a set of externally applied known calibration wrenches

\( \delta \) preload extension applied to the central strut

\( \Delta \) elliptical polar operator having the effect of interchanging primary with dual parts of the screw quantity following

\( \varepsilon \) a set of measured data from the six legs

\( S_i \) unit screw along the \( i \)th leg

\( S_0 \) unit screw along the central strut

1 INTRODUCTION

This paper presents the stiffness-based analysis and synthesis of a new six-component force measuring transducer based on the Stewart platform that can be used to measure the contact forces in robotic grasping and similar applications. The structure of the transducer is similar to that of the well-known Stewart platform manipulator with legs of equal length, and hence symmetrical about its central axis. As shown in Fig. 1, the system consists of a hemispherical tip that is supported by six wire legs and a central compressive strut. External loads to be measured are applied to the surface of the hemispherical tip.

Each of the wire legs is connected to a strain-gauged cantilever forming the local transducer in each of the load paths represented by the legs. The advantage of this configuration is that the contribution to the support of the platform is, as far as possible, only achieved by means of the loads directly measured in the legs, so giving the greatest possible sensitivity. However, with the legs loaded only in tension, a central elastic compressive strut is needed to preload the legs and, as a result, detracts from the overall sensitivity, as the strut will not, to avoid redundancy, be instrumented.

The analysis of the elastic behaviour of the system is developed and exploited to give good sensitivity with respect both to the magnitude and to the direction of the externally measured load. The hemispherical end enables complete information about the nature of the contact forces to be measured, including estimates of the couple exerted about the normal to the surface of contact.

2 BACKGROUND

Force and torque transducers for robot wrist reactions [1, 2], for machining forces [3] and for assembly processes are well known and available commercially. However, most such devices are designed for relatively large force and torque magnitudes, and generally have high cost. Some of these devices are compact, often using a pair of concentric rings connected by strain-gauged double cantilevers. In principle, as demonstrated by the designs of classical machining force measuring systems, strain gauges can be applied to any suitable elastic solid, providing that suitable positions for the gauges can be found. In machining force sensors, it has further been traditional to arrange these positions in such a way that individual components of force and of moment can be detected by specific gauges. This is not theoretically necessary, since, for any elastic solid with sufficient independently positioned gauges, a linear relationship can always be found between the components of any arbitrary externally applied load and the outputs of the gauges.

In most aspects of kinematics and robotics, geometry, particularly line geometry, is one of the key analytical tools, enabling for example, the positioning of an end-effector to be related to the configuration of the robot arm [4]. Similarly, in robotic grasping [5, 6], the lines of action of the forces exerted by fingers, the geometry of the normals and tangents at the contact points on the grasped object and their relation to the shape of the object are of interest in many forms of analysis.

In the validation of any system of theories that describe the distribution [7] of forces in a robotic grasp, a device is required to provide information on the position of the contact point, the orientation of the contact surface, the line of the contact forces and other relevant information such as whether a spin couple exists about the line of the contact normal. The structure of the Stewart platform [8] is a device that, when considered as a candidate for a force and torque transducer, provides many of the desired attributes. As a result of
previous analysis [9], a special arrangement of the six legs presents an elegant way of obtaining suitable stiffness characteristics, and hence sensitivities. The idea of using the Stewart platform as such a transducer was suggested in 1986 [10], when the Stewart platform was proposed as a transducer for cross-coordinate control of robot manipulators. The idea was then developed by means of the study of the elastic properties of such a device [11] with six legs capable of being loaded either in tension or compression. That study presented a global stiffness matrix providing a relationship between the contact forces and the infinitesimal movement of the platform, which was, further, capable of displaying either isotropic translational (or direct) stiffness or isotropic rotational stiffness, but not both.

While there are many methods of producing electrical or electronic signals that are proportional to strain, electrical resistance strain gauges are commonly available and give reliable results providing that established techniques are followed. It is thus important to synthesise a multicomponent force and torque transducer using the elastic properties not only of the structure as a whole but also of the individual transducer elements, namely, in this application, strain-gauged cantilevers. A new structure is hence proposed.

Accordingly, a mathematical model is derived for this new elastic structure; the model is exploited to give advantageous operational behaviour, including the preload required. Calibration is described, and experimental results are given that show the potential application of the transducer.

3 STRUCTURE AND ITS ELASTIC MODEL

The transducer is composed of a base and a platform, being the hemispherical tip on which external forces are to be exerted, and six legs. In order to achieve the maximal sensitivity of each strain gauge in assessing the overall load, it is desirable only to load the legs with axial forces in the direction of the legs, and to avoid bending and torsion.

This is achieved by designing the legs in the form of tensile wires as noted above. Each wire is attached at one end to the platform, and at the other to the free end of a strain-gauged cantilever as shown in Fig. 2a. Two such cantilevers are manufactured together in the V-shaped type shown in Fig. 2b and fixed to the base. These cantilevers intercept the lines of the legs, as in Fig. 2a, representing the Stewart platform geometry. Because of the configuration of the Stewart platform, pairs of legs lie in a plane.

The operation of the system as a force measuring device using strain measurement depends upon the elastic nature of the gauged cantilevers and hence, effectively, of the legs. As a consequence, any device used to pretension the wires must have a stiffness that is not so large that the resultant deflections in the legs are reduced to an extent that the gauges no longer give a useful output. The effect of the stiffness of this central strut is thus included in the overall elastic analysis.

A leg can be described mathematically by a screw along the leg which determines the direction and location of the leg. The form of a screw [12, 13] along the ith leg is given as

$$\boldsymbol{S}_i = [l_i, m_i, n_i, p_i, q_i, r_i]^T$$

with $l, m, n$ forming a unit vector along the leg, and $(p, q, r)$ being the components of the moment of the unit vector about the origin. In this case, the origin is located at the centre of the hemisphere as shown in Fig. 3. Here, $\boldsymbol{S}_i, i = 1, \ldots, 6$, are the screws of zero pitch, that is, line
vectors, along the legs. The force on a leg is hence described by applying a force magnitude to the screw along the leg. The equation of equilibrium of the hemispherical platform under the action of the external wrench, $w_e$, and of the forces provided by the six legs and the central support strut can be written as

$$Jf + f_oS_o + w_e = 0$$  \hspace{1cm} (2)

where matrix $J$ contains the six screws, $S_i$, $i = 1, \ldots, 6$, along the legs, as its columns; $S_o$ is the screw of zero pitch along the central strut; $w_e$ is the external wrench exerted on the platform; $f$ is the six-element vector containing scalars $f_i$, $i = 1, \ldots, 6$, which are the force magnitudes applied along the lines of the legs and $f_o$ is the force magnitude incurred along the line of the central strut.

In the transducer configuration of the platform, base and legs, the six line vectors (screws), $S_i$, along the legs are known to be linearly independent; the screw, $S_o$, along the central strut will thus be linearly dependent upon the six $S_i$. Relating the forces to the axial elastic extensions of the legs and central strut, it follows that

$$f = -Ku$$  \hspace{1cm} (3)

and

$$f_o = -k_o u_o$$  \hspace{1cm} (4)

Diagonal matrix $K$ is the $6 \times 6$ stiffness matrix of the six legs, $k_o$ is the stiffness of the central strut, $u$ is a six-element vector containing the axial extensions of the platform attachment point of the six legs and $u_o$ is the axial extension of the central strut. When the stiffness of each leg is identical, $K$ becomes the identity matrix, multiplied by the scalar, $k_l$, which is the stiffness along the leg. The extensions can also be related to the platform twist $\mathbf{T}$ in screw form. The axial elastic extensions along the six legs are

$$u = J^T \Delta \mathbf{T}$$  \hspace{1cm} (5)

and the axial elastic extension along the central strut is

$$u_o = S_o^T \Delta \mathbf{T}$$  \hspace{1cm} (6)

where the $6 \times 6$ matrix $\Delta$ is the elliptical polar operator [14], interchanging primary and dual parts of the platform twist, $\mathbf{T}$. The relationship between the extensions of the central strut and those of the legs can thus be found by eliminating $\mathbf{T}$ from equations (5) and (6):

$$u_o = S_o^T J^{-T} u$$  \hspace{1cm} (7)

Similarly, the relationship between the forces on the central strut and legs can be given by rearranging equations (3) and (7) as

$$f = -Ku = -KJ^T \Delta \mathbf{T}$$  \hspace{1cm} (8)

Rearranging the above equation and substituting in equation (6) yields

$$u_o = -S_o^T (KJ^T)^{-1} f$$  \hspace{1cm} (9)

It follows from equation (4), assuming the stiffness of each leg is identical and equal to $k_l$, that

$$f_o = -k_o u_o = k_o S_o^T (KJ^T)^{-1} f = \frac{k_o}{k_l} S_o^T J^{-T} f$$  \hspace{1cm} (10)

thus giving the relationship between the forces incurred in the six legs and that in the central strut. Combining equation (10) with equation (2), the modified equilibrium equation is

$$w_e = -\left( J + \frac{k_o}{k_l} S_o S_o^T J^{-1} \right) f = J_A f$$  \hspace{1cm} (11)

Matrix $J_A$ is the modified equilibrium matrix augmented with the contribution from the central strut, thus giving a direct linear relationship between the external wrench to be detected and the forces in the six legs in the new device without referring to the force element in the central strut. As expected, it contains the ratio of the stiffness of the strut to that of the legs, as well as information relating to the structural geometry of the device.

4 GLOBAL STIFFNESS AND RELATIONSHIP BETWEEN THE WRENCH AND INFINITESIMAL TWIST

The relationship between the wrench and infinitesimal twist provides a $6 \times 6$ global stiffness matrix for the
device. It can be shown, by rearranging equation (8) and
the augmented equilibrium equation (11), that

\[ \mathbf{w}_c = (\mathbf{JKJ}^T + \mathbf{k}_o \mathbf{S}_o \mathbf{S}_o^T) \mathbf{\Delta T} = \mathbf{K}_A \mathbf{\Delta T} \quad (12) \]

The global stiffness matrix is hence given in an aug-
mented form. The first part is the sum of the trans-
formed leg stiffness matrices reflecting the coordinate
transformation from the local frames to the global. It is

\[ \mathbf{JKJ}^T = \frac{k_1}{a^2 + b^2 - ab + h^2} \begin{bmatrix} c_1 & 0 & 0 & c_2 & 0 \\ 0 & c_1 & 0 & -c_2 & 0 \\ 0 & 0 & 6h^2 & 0 & 0 \\ 0 & -c_2 & 0 & 3b^2h^2 & 0 \\ c_2 & 0 & 0 & 0 & 3b^2h^2 \\ 0 & 0 & 0 & 0 & \frac{9b^2a^2}{2} \end{bmatrix} \quad (13) \]

where \( c_1 = 3(a^2 - ab + b^2) \) and \( c_2 = 3bh(a - 2b)/2 \). The second part is the effect of the central strut and is

\[ \mathbf{k}_o \mathbf{S}_o \mathbf{S}_o^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{k}_o & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (14) \]

The augmented stiffness matrix, \( \mathbf{K}_A \), can be partitioned
into 3 \times 3 submatrices. The first diagonal submatrix
gives the direct or translational stiffness, and the second
diagonal submatrix gives the torsional stiffness. The other two off-diagonal submatrices represent crosslink
terms between forces and couples, and rotation and trans-
lation. Hence, the relationship between the wrench
and twist is not only dominated by direct link relationships
between forces and translations, and between couples and rotations, but also by their crosslinks.

The measured strains are directly related to the plat-
form elastic twist, \( \mathbf{T} \). The augmented stiffness matrix,
\( \mathbf{K}_A \), relates this twist to the externally applied load. It is
clearly desirable that the device as a whole should be as
equally sensitive as possible to loads of all kinds and
from all directions. The existence of off-diagonal terms
in \( \mathbf{K}_A \) will hinder this objective, as will significant
inequalities between the diagonal terms, taking the
direct and rotational entries as separate entities.

The design of the device should be such as to have no
off-diagonal terms in \( \mathbf{K}_A \). This will also simplify analysis
and can be achieved by setting \( a = 2b \) so that \( c_2 \) in
equation (13) vanishes.

Hence, the external wrench, \( \mathbf{w}_c \), and the associated
elastic infinitesimal strain \([15] \), \( \mathbf{T} \), of the platform are
related by means of a diagonalized simplified form.
Equations (12), (13) and (14) show that there is now
considerable choice in the range of parameters available
in the design.

5 EFFECT OF LOCAL STIFFNESS ON GLOBAL
STIFFNESS

The analysis leading to the global stiffness matrix pre-
sents a basis for design relating to the stiffnesses of the
legs and the central strut. In particular, the linear relation-
ship between the two stiffnesses plays a key role in
the design of the device. In considering these stiffnesses,
there are two attractive alternatives: isotropic torsional
stiffness and isotropic direct stiffness.

Isotropic torsional stiffness can readily be achieved
from the above stiffness matrix by arranging the geo-
metric properties of the platform to satisfy

\[ h = \sqrt{6}b \quad (15) \]

In this case, the last three diagonal elements of the
stiffness matrix become equal. The result is similar to
this particular configuration, the addition of a central
strut in the Stewart platform-based device does not
affect the torsional stiffness. This can be perceived from
the augmented stiffness matrix in equation (14).

However, adding a central strut presents a new pos-
sibility for isotropic direct stiffness. To achieve this,
equating the third diagonal stiffness element to each of
the first two and letting \( a = 2b \) as noted above, the
following is obtained:

\[ \frac{6h^2k_1}{3b^2 + h^2} + k_o = \frac{9b^2k_1}{3b^2 + h^2} \quad (16) \]

Rearranging the above yields

\[ \frac{k_o}{k_1} = \frac{9 - 6(h/b)^2}{3 + (h/b)^2} \quad (17) \]

Isotropic direct stiffness is thus provided with the above
relationship, which is shown in Fig. 4.

It is clear in equation (17) that, in the absence of the
central strut, isotropic direct stiffness occurs when the
platform geometry ratio \( h/b \) is equal to \( \sqrt{1.5} \). Isotropic
torsional stiffness demands that \( h/b \) is equal to \( \sqrt{6} \).
However, the resulting stiffness ratio, \( k_o/k_1 \), becomes
negative. Thus, it is not practically feasible to achieve
both isotropic direct stiffness and isotropic torsional stiffness: a compromise must therefore be found.

The stiffness, \( k_l \), of each leg is determined largely by the tip stiffness of the cantilevers for the strain gauges. For an effective cantilever length, \( L \), width, \( b \), and thickness, \( t \), the tip stiffness is

\[
k = \frac{Ebt^3}{4L^3}
\]

and the normal strain, \( \varepsilon \), at a point near the root of the cantilever is

\[
\varepsilon = \frac{6fL}{Ebt^2}
\]

The tensile stiffness of the wire connecting the platform to the base is, by comparison, typically one-tenth of the cantilever stiffness. Although this modifies the effective leg stiffness, the effect of this is dealt with, for modelling purposes, in the stiffness of the legs.

### 6 PRELOAD DESIGN

The elastic, compressively loaded central strut is used to keep the legs in tension. The strut is composed of a spring, two ball joints which connect the ends of the spring to the platform and an adjusting screw mounted on the base. The preload is achieved by changing the displacement \( \delta \) in the spring to ensure that the legs are always in tension when subjected to the expected range of external loads. The preload force in the central strut is

\[
f_{p0} = k_o(\delta - u_{p0})
\]

The resulting forces of the six legs owing to this preload are a six-element vector, \( f_p \), which forms an equilibrium equation as follows with the preload only:

\[
Jf_p + f_{p0}S_o = 0
\]

Substituting the displacement equations (3) and (20), the above equilibrium equation becomes

\[
-JKu_p + k_o(\delta - u_{p0})S_o = 0
\]  

Further, substituting equation (7) in the above equilibrium equation owing to the preload yields

\[
-JKu_p + k_oS_o(\delta - S_o^TJ^{-1}u_p) = 0
\]

and hence

\[
\begin{align*}
\delta & = S_o^TJ^{-1}u_p \\
& = S_o^TJ^{-1}k_oS_o
\end{align*}
\]

From equations (5) and (12), the extensions of the legs owing to external load are given by

\[
\delta = \frac{1}{k_o}S_o^Tw_e
\]

It is required that the force magnitudes in the six legs owing to preload should be larger than those incurred by an external wrench, to keep the legs in tension. That is

\[
f_p > f
\]

Each element of \( f_p \) must be greater than the corresponding element of vector \( f \). The above condition can be rearranged by using equations (3) and (25) as

\[
\begin{align*}
\delta & > \frac{1}{k_o}S_o^Tw_e \\
& > \frac{1}{k_o}k_oS_o
\end{align*}
\]

In the special case of the geometry of this device, this inequality can be reduced to

\[
\delta > \frac{1}{k_o}S_o^Tw_e
\]

It can clearly be seen from equation (28) that the worst case is when \( w_e \) is a force applied along the \( z \) axis of the device. Hence, the preload design is related to the designated external wrench which can be selected as a maximum external load on the transducer. The transducer is set with a designed preload.

### 7 TRANSDUCER OVERVIEW

In the light of the above analysis, including desirable levels of strain under the gauges, and using standard materials and components, the following values for design were chosen:

- Pitch radius of base leg mounts on base, \( a \) 40 mm
- Pitch radius of platform leg mounts on platform, \( b \) 20 mm
Equivalent height of platform, $h$ 49 mm  
Width of cantilever, $b_t$ 10 mm  
Thickness of cantilever, $t$ 1.46 mm  
Length of cantilever, $L$ 19.8 mm  
 Tilting angle of V-shaped double cantilevers, $\theta$ 30°  
Outside diameter of central strut, $d_c$ 15 mm  
Stiffness of central strut, $k_o$ 270.3 N/mm  
Stiffness of leg, $k_l$ 97.2 N/mm  
Thickness of wire in leg, $t_w$ 0.37 mm  
Nominal length of leg, $l$ 60.0 mm

8 CALIBRATION AND TRANSDUCER SYSTEM

The geometry of the force transducer is incorporated in the augmented transducer matrix, $J_A$, in equation (11). However, this matrix is dominated by the geometry of the device. Manufacturing and assembly errors inevitably affect the accuracy of the measured results; the gain factors from the amplifiers, data logger and D/A converter also affect the results. In practice, these effects can be dealt with by means of an appropriate calibration process by introducing a $6 \times 6$ calibration matrix, $C_A$, as follows:

$$w_e = C_A \varepsilon$$  (29)

where $w_e$ is the external wrench to be calculated and $\varepsilon$ is a six-element vector aggregated from the strain gauge signals of the six legs. Hence, this calibration matrix includes all the factors relating to the strain gauges and the manufactured geometry.

To identify matrix $C_A$, a modified least square method was used. By aggregating a set of $n$ known trial wrenches in the columns of the $6 \times n$ matrix $W$, and a set of corresponding data output from the strain gauges as columns of the $6 \times n$ matrix $E_s$, the following formula gives the calibration matrix:

$$C_A = W E_s^T (E_s E_s^T)^{-1}$$  (30)

To ensure the measuring data matrix $E_s E_s^T$ is invertible, it is necessary that, among the selected external trial wrenches, six are independent. Hence, the calibration starts with a number, $n$, of known external loads applied through suitable, known attachment points that allow both tensile and compressive forces to be exerted on the hemispherical tip. A screw system [16] was used to arrive at a proper number of trial wrenches. Details of such procedures will be described elsewhere.

For the purpose of the calibration, the transducer was mounted on a simple fixture and at a number of different orientations. The calibrated matrix was then stored in a versatile computer software package, the flow chart of which is shown in Fig. 5. The software [17] was developed to evaluate the signals from a set of strain gauges on the V-shaped cantilevers connecting to a data logger.

The measured, calibrated values of the elements of $C_A$ are as follows:

$$

calculated external wrench, $w_e$. This wrench consists of the external force, $f_e$, as its primary part, and the total moment, $m_e$, about the origin of all applied loads as its secondary part. Decomposition of this wrench reveals its pitch, $p_h$, and the perpendicular position, $r_o$, of the line of action of force $f_e$ according to

$$p_h = \frac{f_e \cdot m_e}{f_e \cdot f_e} \quad \text{and} \quad r_o = \frac{f_e \times m_e}{f_e \cdot f_e}$$  (32)

From consideration of the geometry of the tip, the actual position of the point of application of the load can be shown to be

$$r = r_o - \alpha f_e$$  (33)

where $\alpha = \sqrt{b^2 - ||r_o||^2}$ [18]. From this, the unit vector, $\hat{r}$, of the normal at that position is easily found. Also, the normal component of the contact force, the vector form of the tangential force, and the component of total moment about the normal can be found [18].

A number of practical tests were carried out to verify and illustrate the performance of the device. Three of these are reported here; firstly, to test the force measurements; secondly, to assess the tangential frictional component of a contact force; and finally, to use the device as a means of detecting contact with an unknown object and to secure geometrical information about the contact with that object.

9.1 Measurement of a known external load

The sensor was mounted as shown in Fig. 6. A load of 21.82 N was suspended as in the figure from a jig-bored eyelet mounted on the surface in such a way that the effective position of application of the load was $r =$
The nominal external wrench, \( w_{nm} \), associated with this load is

\[
 w_{nm} = (-15.43 \quad 0.00 \quad 15.43; \\
 0.00 \quad -309.00 \quad 0.00)^T \text{N; N mm} \tag{34}
\]

After appropriate loading, the measured strains, \( \varepsilon \), were converted into a measured wrench \( w_e \) by means of calibration equation (29) to give

\[
 w_e = (-15.58 \quad 0.10 \quad 15.78; \\
 -2.55 \quad -303.90 \quad -1.86)^T \text{N; N mm} \tag{35}
\]

Using the standard formulae for the decomposition of screws, wrench \( w_e \) yields force

\[
 f_e = (-15.58 \quad 0.10 \quad 15.78)^T \tag{36}
\]

with a magnitude of 22.18 N and pitch

\[
 p_h = -0.04 \text{ mm} \tag{37}
\]

The perpendicular position vector, \( r_o \), of its line of application is

\[
 r_o = (9.73 \quad -0.14 \quad 9.61)^T \text{mm} \tag{38}
\]

Since the load was suspended from the hemispherical tip of the sensor, equation (33) becomes

\[
 r = r_o + \alpha f_e \tag{39}
\]

Hence, with the known radius of the hemispherical tip, the calculated position of the line of application on the surface of the hemisphere is

\[
 r = (-0.47 \quad -0.07 \quad 19.99)^T \text{mm} \tag{40}
\]

and the magnitude of the couple parallel to the calculated direction of the external force is

\[
 |p_h f_e| = 0.89 \text{ N mm} \tag{41}
\]

The relevance and accuracy of the calculated results may be appreciated by comparing the magnitudes of the external loads (21.82 N applied, 22.18 N measured) and the angles of the action line and normal between nominal and measured (the error in direction cosine of \( x, y \)
and $z$ is 0.5, 0.3 and 0.4° respectively). This can also be appreciated by comparing the position of the calculated load point, where the error is $e = (-0.44, -0.07, 0.01)^T$ mm, and the calculated magnitude of the parallel couple, 0.89 N mm, which should be zero for ideal accuracy.

### 9.2 Slip test

A small metal object was placed upon the crown of the transducer and the sensor was tilted, rotating about a horizontal axis until the block just began to slip. The mass was 31.8 g (weight 0.312 N). The results, presented in the same manner as in Section 9.1 above, are as follows:

$$w_e = (-0.11, -0.08, -0.29);$$

$$0.61 \quad 1.36 \quad -0.56)^T \text{N; N mm} \quad (42)$$

The external force $f_e$ is given by the first three elements of the above measured wrench, with a magnitude of 0.32 N, and the pitch in the measured wrench $w_e$ is

$$p_h = -0.07 \text{ mm} \quad (43)$$

The perpendicular position vector, $r_o$, of the action line of the force is

$$r_o = (4.29, -2.27, -0.85)^T \text{mm} \quad (44)$$

and, by means of equation (33), the position vector of the contact point is

$$r = (10.35, 2.58, 16.71)^T \text{mm} \quad (45)$$

and the magnitude of the couple parallel to the calculated direction of the external force is

$$|p_h f_e| = 0.02 \text{ N mm} \quad (46)$$

Since the direction of the contact normal is

$$\hat{n} = (0.52, 0.13, 0.84)^T \text{mm} \quad (47)$$

the force along the contact normal is

$$f_n = (f_e \cdot \hat{n}) \hat{n} = (-0.18, -0.05, -0.26)^T \text{N} \quad (48)$$

and thus the component tangential to the contact surface is

$$f_t = f_e - f_n = (0.07, -0.03, -0.03)^T \text{N} \quad (49)$$

The magnitude of the normal force is 0.32 N, the magnitude of the tangential force is 0.08 N and the magnitude of the spin couple about the surface normal is 0.02 N mm. The ratio of tangential to normal forces at the point of slip is 0.25.

The quality of these results can be assessed from the following data derived from the results: the magnitude of the externally applied force with the effect of the mass was 0.32 N, and the radius of the hemisphere was 19.82 mm. With a normal force applied on the top of the hemisphere, the error in detecting a position vector was about 0.18 mm, and the calculated magnitude of the parallel couple was 0.02 N mm, which should be zero for ideal accuracy.

### 9.3 Contact test

In this case the sensor was mounted, with its $z$ axis horizontal, on a travelling block so that the sensor could be advanced along a direction parallel to the $z$ axis, without rotation. A hard vertical surface was arranged, making different angles to the line of the advancing sensor in order to obtain a set of results. When contact was first made, the on-line software revealed the following results, presented in the same format as in Section 9.1:

$$w_e = (-0.06, 0.01, 2.24;$$

$$-0.14, -0.01, -0.02)^T \text{N; N mm} \quad (50)$$

$$p_h = -0.01 \text{ mm} \quad (51)$$

$$r_o = (0.00, -0.06, 0.00)^T \text{mm} \quad (52)$$

and so

$$r = (-0.06, -0.01, 19.99)^T \text{mm} \quad (53)$$
which makes the normal of the contacting surface

\[ \hat{n} = (-0.00 \ 0.00 \ 1.00)^T \text{mm} \]  

(54)

Hence, the surface angle detected is 89.9°. More results were carried out on different surface angles and are shown in Table 1. The system can be used to detect the presence of an object in contact and the relative geometry of the surface of that object.

### 9.4 Errors and comments

The accuracy of measurements by this transducer may be gauged from the nature of the results obtained above. The calibration procedure described in Section 8 leads to a level of errors in the estimation of the applied external wrench, \( \mathbf{w}_e \), that is itself of the same order of magnitude as the discrepancy between nominal and measured values. Normal calibration of strain gauges can lead to an accuracy of 10\( \mu \text{e} \), which, in terms of the calibration through equation (31), may lead to errors in the direction of the typical forces, whose measured strain is 500\( \mu \text{e} \). A typical error force may thus be

\[ \delta f_e = (-0.01 \ 0.01 \ 0.04)^T \text{N} \]  

(55)

which leads to a typical error in the direction of a 21.82 N force of 0.5°. The error in calculated surface normal direction is negligible, with error appearing in the third decimal place. These errors are consistent with those observed in the above tests.

The mass of the sensor hemisphere tip is approximately 0.05 kg, and a typical stiffness associated with the leg support system is 200 N/mm. This leads to a typical natural frequency of the order of 320 Hz. For the intended applications of this sensor, dynamic effects are therefore not significant. Long-term stability is dependent on strain gauge stability and has not so far given rise to difficulty.

The geometrical design of the transducer and hence its elastic and sensor qualities have resulted in a practicable system that is capable of measuring a number of features of the contact forces acting between two bodies. In particular, the preliminary tests reported here suggest a measuring device appropriate to robotic grasping experiments and other sensing applications where physical contact is involved. The results of grasping experiments themselves are the subject of another publication.

### 10 CONCLUSIONS

The modelling, analysis and design of a novel contact force and torque transducer have been discussed, and the effect of a central, tensioning strut has been defined. The addition of this new central strut to a Stewart platform-based transducer results in a six-component force transducer, where six legs are replaced with wires to transmit tensile forces only to increase the accuracy, and with three V-shaped strain-gauged cantilevers.

A new elastic model based upon the addition of a central strut has been developed. This produces a relationship for the ratio of the stiffness of the legs to that of the central strut, including the geometry of the new device. The resulting design has addressed issues of choice of extrinsic elastic properties. By means of suitable representative tests, it has been shown that this design is capable of yielding important information on the nature of contact forces found in robotic grasping.

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