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Capital Taxes, Labor Taxes and the Household

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Abstract

We study the impact of capital and labor taxation in an economy where couples bargain over the intrahousehold allocation. We present a life cycle model with heterogeneous individuals and incomplete financial markets. Drawing from the literature of the collective framework of household behavior, we model decision making within the couple as a contract under limited commitment. In this framework more wealth improves commitment and gives rise to insurance gains within the household. Our theory motivates these gains by the empirical observation that wealth, in contrast to labor income, is a commonly held resource within households. Based on this observation we study whether eliminating capital taxes from the economy, and raising labor taxes to balance the government’s budget, may generate welfare gains to married households. We illustrate that the quantitative effects from this reform are rather small. We attribute the small effects to the life cycle pattern of wealth accumulation and to the impact of labor income taxes on household risk sharing: In particular, we show that higher labor taxes may deteriorate the limited commitment problem, even though they may make the distribution of labor income more equitable within the household.

JEL codes: D13, D52, E21, E62, H31
Keywords: Life cycle models, incomplete financial markets, tax reform, intrahousehold allocations

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1 Introduction

A recent literature in dynamic quantitative macroeconomics studying the effects of taxes in models of heterogeneous ‘bachelor’ households has found large welfare gains from positive capital taxation. For example, Conesa et al. [2009] illustrate that a high tax rate on capital is optimal for young households. Domeij and Heathcote [2004] illustrate that eliminating capital taxes would benefit only a small fraction of wealthy families, while imparting large welfare losses to the rest of the population.

Given that these conclusions are based on models where the wellbeing of the household coincides with the wellbeing of a single individual, typically the male household head, they represent an accurate approximation of the between household inequality and distributional effects, but ignore a potentially important analogous effects at the intrahousehold level. It is, for example, conceivable that a policy change which sets capital taxes to zero or to a lower level than the current one, and raises labor taxes to cover for the loss in revenues, on the one hand exacerbates the between household inequality, and on the other, makes the distribution of resources more equitable within households. In such cases it is obvious that a more explicit modelling of the intrahousehold decision making process is needed.

In this paper we advocate that the available evidence, presented in Mazzocco et al. [2007] and discussed extensively in section 2 of our paper, concerning the joint decisions of couples in the US, supports this hypothesis. In particular, the evidence suggests that the following two assumptions are accurate with regards to the joint decision making process: First, wealth is a commonly held (pooled) resource within the family and second, (individual) labor income is not. Based on this observation we investigate how tax policies, by influencing the relative importance of wealth and labor income on the household’s budget, may affect the intrahousehold allocation. In our experiment, we consider a policy change whereby the government eliminates capital taxation and raises labor taxes to balance the budget. Our key finding is that though the qualitative impact of the policy change is one which is likely to reduce the intrahousehold inequality, in quantitative terms the effects from policy are rather small.

Our conclusion is drawn from a dynamic model where households (couples) live for several periods and make savings and labor supply decisions. They experience shocks to their labor income but are unable to fully hedge against them. Imperfect insurance derives from the fact that households face borrowing constraints in the economy, but also because risk sharing is imperfect within the household. To model incomplete intrahousehold insurance we add the following ingredient: We assume that the intrahousehold allocation is the outcome of a limited commitment contract whereby the two spouses maximize joint welfare according to a sharing rule which reflects their bargaining power. When the labor income of one spouse increases, the sharing rule allocates more resources to her, implying that the couple has to give up some risk sharing to satisfy a participation constraint. This feature of our model represents a dynamic extension of the literature in the collective model (see for example Chiappori [1988, 1992], Apps and Rees [1988], and Blundell and Etheridge [2010]) broadly similar to the models considered by Ligon et al. [2000], Mazzocco et al. [2007] and Voena [2012]. As in Mazzocco et al. [2007] and Voena [2012] we assume that the bargaining power is determined by the value that the spouses would get if they were to divorce. However, we do not allow divorces to occur in equilibrium. We impose this feature to our model because we wish give it the possible best chance to yield a substantial improvement from policy. Divorce is practically a breakdown of commitment,
i.e. it limits the scope and the horizon over which intrahousehold insurance is relevant.

In section 3 of our paper we use a simplified version of the theory (a two period model) that enables us to derive analytically the effect of the tax schedule on the intrahousehold allocation. Our results are as follows: First, lower capital taxes are shown to improve commitment and risk sharing within the household. Our analysis shows that as the household’s financial income increases, and a larger fraction of consumption is financed through wealth, the participation constraints and the limited commitment friction are relaxed. Second, labor taxes have an ambiguous effect on intrahousehold risk sharing: On the one hand, higher labor taxes reduce intrahousehold inequality in income and therefore improve the household’s insurance possibilities, but through a second channel they tighten the participation constraint, as they impoverish the household and make it more tempting to re-bargain. The latter effect is more important the higher is the degree of risk aversion of individuals, and at low levels of household savings.

In the quantitative life cycle model the balance of these effects varies across age cohorts. Young individuals have little wealth and therefore are more likely to be adversely affected by the rise in labor taxes. In contrast, middle aged and older households accumulate sufficient wealth and they can benefit from the change in both capital and labor taxes. Moreover, in the dynamic life cycle context, the severity of limited commitment friction varies with the stage of the life cycle. Typically, shocks which occur very early on in the life cycle are less persistent and have a modest effect on the participation constraints. Shocks experienced later on in the life cycle are more significant.

When the reform takes place, there are only minor effects accounted for by a few cohorts. The reason is that households which stand to gain most from the change in policy, the middle-aged and older households, are also households which have accumulated sufficient wealth, even prior to the elimination of capital taxes; for these households the limited commitment friction is less severe, even in the original steady state, and a policy which encourages further wealth accumulation does not improve welfare significantly. Overall, we find that re-bargaining is less frequent, however the gains from risk sharing, measured in terms of lower consumption variability are very modest. When we look at the welfare effects of the reform we establish that married individuals experience gains in the final steady state which are well within the range of the analogous gains experienced by ‘bachelor’ households. By and large our conclusion is that when considering to set the level of capital taxation, the government does not have to worry about large distributional effects within households.

This paper is related to several strands in the literature. First, there is considerable literature on the intrahousehold allocation within the so called collective framework (see for example Chiappori [1988, 1992], Blundell and Etheridge [2010]). Our assumption that family members do not pool labor income and that changes in the labor income of individuals lead to an updating of the sharing rule is based on this literature. Many papers have used the collective framework to investigate the properties of optimal taxation with particular focus on whether tax schedules should be different between men and women in light of the differences in the elasticity of labor supply (for example Apps and Rees [1999], Apps and Rees [2011] and Alesina et al. [2011]). Though this analysis is clearly important and bears significant effects on the welfare of families, the models are typically static and therefore not suitable to analyze how the tax schedule affects the intertemporal behavior of couples. Our approach is to abstract from the complexities of the tax code and to summarize the institutions in a simple linear tax system. But since our model is dynamic, we add to the literature
by formalizing how changes in linear taxes can affect the welfare and the risk sharing possibilities of families.

As discussed previously, dynamic life cycle models of intrahousehold bargaining have been considered by Mazzocco et al. [2007] and Voena [2012] (along the lines of the infinite horizon model of Ligon et al. [2000]). Our model is similar to theirs, however, the aspects of the intrahousehold allocation which we study are not the focus of either one of these papers. In particular, in Mazzocco [2007] and in Mazzocco et al. [2007], differences in risk aversion between the male and the female spouse are the key channel through which the bargaining process affects intertemporal behavior. Voena [2012] studies the effect of changes in divorce laws on the intertemporal behavior of families and shows that when unilateral divorces were institutionalized, the saving behavior of couples changed significantly. Our results, which show that the impact of taxes is modest, should not be misconstrued to mean that studying the dynamic aspects of intrahousehold bargaining is not important; Mazzocco et al. [2007] and Voena [2012] both make a strong case to the contrary, focusing on different aspects of intrahousehold decisions than we do.

There is a sizable literature on the optimality of the US tax code in models of heterogeneous agents and wealth accumulation (for example Aiyagari [1995] and Imrohoroglu [1998] and Domeij and Heathcote [2004], Conesa et al. [2009] and Conesa and Krueger [2006]) which for its most part has relied on the bachelor household framework. In this framework household wealth is important as a buffer against labor income shocks. Our analysis identifies an additional margin through which families may benefit from wealth, due to its influence on the intrahousehold allocation. That is to say wealth in our framework, contributes towards mitigating the effects of idiosyncratic shocks by promoting efficient risk sharing within the household, but also through enabling the household to self-insure against the aggregate component of these shocks.

Finally, in recent work of Guner et al. [2012a,b] consider dual earner households and document considerable efficiency gains from (revenue neutral) reforms which replace joint filling with separate filling. These gains derive mainly from the response of female participation in the labor market. Our paper also presents a model with gender and marital status heterogeneity, but relative to Guner et al. [2012a,b] who consider a unitary model, with no shocks to the labor income of individuals, our focus is on the effects of policy changes on the intrahousehold allocation and on risk sharing in a non-unitary model and in the presence of idiosyncratic income shocks.

The paper proceeds as follows: Section 2 motivates our assumptions concerning the treatment of wealth within the family, by reviewing recent empirical evidence. Section 3 studies the effects of taxes in the two period version of our theory. Section 4 presents the quantitative model. Section 5 discusses the results from the reform. A final section concludes.

2 Wealth Pooling

Our starting point in this paper is to assume that wealth, in contrast to labor income, is a commonly held resource in the household. The motivation for this assumption mainly derives from the institutional framework governing divorce in the US, and the relevant empirical evidence on divorce

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1The evidence against labor income pooling are well documented in the considerable literature on the intrahousehold allocation (see for example Blundell and Etheridge [2010]).
settlements. The evidence to which we refer is presented in Mazzocco et al. [2007] and Voena [2012]. We provide here, a summary of the observations.

There are three different property laws in the United States: common property, community property, and equitable property law. Common property establishes that marital wealth, during divorce, is divided according to who has legal title to the property. Only the state of Mississippi has common property law. In the remaining 49 states, all wealth acquired during the marriage is considered community property and divided either equally between the spouses in community property states, or equitably in equitable property states. In the latter case of equitable division, courts take into account a variety of factors (such as individual labor income, fairness, etc.) when they determine the fraction of marital property that is allocated to the spouses. The empirical estimates presented in Mazzocco et al. [2007] are indicative of the distributions of equitable outcomes. They estimate, from a sample of divorce settlements from the National Longitudinal Study of the High School Class of 1972 (NLS-72), that the average percentage of wealth allocated to the wife is 0.496, with a standard deviation of 0.177.

Based on the above remarks we construct a theoretical model with the following two properties in our treatment of wealth: First, in the household’s program, we treat wealth as a common state variable, meaning that we do not allow individuals to save in private accounts, but rather force them to pool all wealth within the household. Second, we treat as our benchmark the case where in divorce, each spouse gets 50% of all savings accumulated during the marriage. The second assumption is mainly a simplification of the environment which enables us to derive analytical results (in section 3). We will show however that our results are robust to assuming that the division of assets in the case of divorce is random, with a mean of one half (as the data suggests). The first assumption, that wealth is a common state variable, is in our framework an accurate approximation for several reasons: First, because all wealth in the model is wealth accumulated during the marriage. Second, because in our model the intrahousehold allocation is pinned down by two (Pareto) weights which are determined by the outside options of individuals. Even if individuals are allowed to save separately, insofar as the intrahousehold allocation entails transfers between the spouses, and these transfers reflect the Pareto weights, saving commonly in the household, or saving separately as individuals, should be irrelevant.

Finally, we note that the assumption of jointly held savings is rather common in quantitative macroeconomic models (see for example Mazzocco et al. [2007], Guner et al. [2012a,b], and Cubeddu and Ríos-Rull [1997] among others).

3 Intrahousehold Bargaining in two Periods

This section presents a simplified version of our theory. It investigates the optimal behavior of a couple which consists of a male and a female spouse, lives for two periods, but faces the limited commitment problem only in period two. We derive analytical results that establish the impact of the tax schedule on the intrahousehold decision process.\(^3\)

\(^2\)This holds unless the spouses agree that certain assets are separate property.

\(^3\) We embed the limited commitment friction in our model basically following the approach of Ligon et al. [2000]. In that paper the authors derive the limited commitment friction from a constrained efficient optimization program. We refer the reader to that paper for the more rigorous treatment, and describe here the intrahousehold bargaining
We assume that preferences of both the male and the female spouse, are represented by \( u(c_g^t, l_g^t) \) with the properties \( u_c > 0, u_l > 0, u_{cc} < 0, u_d < 0 \) where \( c_g^t \) denotes consumption and \( l_g^t \) is leisure in period \( t = 1, 2 \). The superscript \( g \) denotes gender ( \( g \in \{m, f\} \) (male , female) ). Moreover, we assume each household member enjoys a constant utility flow \( \xi > 0 \) each period from being married. Following Voena [2012] we label \( \xi \) an affection parameter which (in our model) both spouses enjoy from being together. However, it should be understood that \( \xi \) may equally represent (in reduced form) a benefit accruing to married households from complementarity in the production of a home good which is not modelled here.\(^4\) If the marriage breaks up we set \( \xi \) equal to zero.

Our analysis in this section, is derived in partial equilibrium. That is to say, we state our results keeping the wage rate \( w \) and the interest rate \( r \) constant. Moreover, we assume that there is a government which levies capital and labor taxes, denoted by \( \tau_K \) and \( \tau_N \) respectively. The household is born with level of wealth \( a_0 \), and accumulates wealth in period 1 to finance consumption in \( t = 2 \). The household’s members are assumed to be identical initially in terms of productivity (normalized to unity for simplicity), but in period two each individual experiences a shock in their productivity endowment, that alters their labor income potential. We let \( \epsilon_g \) be the productivity for the household member of gender \( g \) at \( t = 2 \).

The limited commitment problem emerges from the fact that differences in productivity across the two members affect the relative bargaining power within the household, and thus affect the intrahousehold allocation. To characterize this allocation we assign a weight to the welfare of each individual in the household. We let \( \lambda_t \) be the share of the male spouse for \( t = 1, 2 \) and \( 1 - \lambda_t \) the analogous share of the female spouse. The fact that \( \lambda_t \) is time dependent reflects the lack of commitment in our model. For instance, we envisage that if in period two, the realized value of \( \epsilon_m \) is large enough, the male spouse may command a larger share of household resources, in which case there will be a renegotiation of the value of his share upwards. To be more precise, the couple inherits a sharing rule \( \lambda_1 \) from period 1. If the male spouse needs to be made better off then the rule will settle to a new value \( \lambda_2 > \lambda_1 \). We assume, following Ligon et al. [2000] and Mazzocco et al. [2007], that this new value \( \lambda_2 \) must be such that the male spouse is as well off as they would be if the marriage broke up.\(^5\) Analogously, if the female spouse has to be made better off, then \( \lambda_2 < \lambda_1 \). If participation is not violated for either spouse, then \( \lambda_2 = \lambda_1 \).

The household contract is represented as a joint program, given these weights. We let \( M_1(a_0, \lambda_1) \) be the value function that characterizes this joint maximization problem in period 1 and \( M_2(a_1, \lambda_2, \epsilon) \) the analogous function for \( t = 2 \). Notice that the latter has \( \epsilon \) as an argument which now represents the vector of idiosyncratic productivities in period 2. To characterize the household’s program we solve backwards. Given the wealth endowment and the levels of productivity, \( M_2(a_1, \lambda_2, \epsilon) \) is a solution to:

\[
M_2(a_1, \lambda_2, \epsilon) = \max_{c_2^g, l_2^g} \lambda_2 u(c_m^2, l_m^2) + (1 - \lambda_2) u(c_f^2, l_f^2) + \xi
\]

process in a simplistic way.

\(^4\)In the Appendix we show that our analysis can be extended to the case of a home good whose consumption is public to the household.

\(^5\)See also Gallipoli and Turner [2011] and Voena [2012].
subject to:

\[
\sum_g c^2_g = \sum_g (1 - l^3_g)w(1 - \tau_N)\epsilon_g + a_1(1 + r(1 - \tau_K))
\]

Standard results imply that the optimal allocation satisfies \( \lambda_2 u_c(c^m_2, l^m_2) = (1 - \lambda_2)u_c(c^f_2, l^f_2) \) and \( \frac{u_l(c^g_2, l^g_2)}{u_l(c^g_1, l^g_1)} = w(1 - \tau_N)\epsilon_g \). To introduce formally the notion that the contract maybe rebargained we let \( S_g(D_g(a_1), \epsilon_g) \) be the utility of the household member of gender \( g \) if the marriage breaks up, where \( D_g(a_1) \) is a division of family wealth in the event of divorce with the property \( \sum_g D_g(a_1) \leq a_1 \). The participation constraints which must be satisfied are:

\[
u(c^g_2, l^g_2) + \xi \geq S_g(D_g(a_1), \epsilon_g) \quad g \in \{m, f\}
\]

The renegotiation of the marital contract takes the following form:

\[
\lambda_2 \in \arg \min_{\lambda^*} |\lambda^* - \lambda_1| \quad \text{such that} \quad V_g(a_1, \lambda^*, \epsilon) \geq S_g(D_g(a_1), \epsilon_g) \quad g \in \{m, f\}
\]

where \( V_g \) is the level of utility of the household member of gender \( g \) that derives as a solution to program (1) if the weight is equal to the value \( \lambda^* \). (3) says that the share is updated in those states where the participation constraint is violated, and is otherwise constant. Whenever there is a change in \( \lambda_2 \) relative to \( \lambda_1 \), this change is the minimum required to satisfy participation for both spouses.\(^6\)

Given the above derivations, we can represent the household’s program in period 1 as follows:

\[
M_1(a_0, \lambda_1) = \max_{c^1_1, l^1_1} \lambda_1 u(c^m_1, l^m_1) + (1 - \lambda_1)u(c^f_1, l^f_1) + \xi + \beta \int M_2(a_1, \lambda_2, \epsilon)dF(\epsilon_m, \epsilon_f)
\]

subject to:

\[
\sum_g c^g_1 + a_1 = \sum_g (1 - l^3_g)w(1 - \tau_N) + a_0(1 + r(1 - \tau_K))
\]

where \( \beta \) is the discount factor of the household, and \( F(\epsilon_m, \epsilon_f) \) represents the joint cdf of productivity shocks in the family. Note that given the individuals are identical in the first period we can, without loss of generality, set \( \lambda_1 = \frac{1}{2} \).\(^7\)

### 3.1 The Effect of Taxes on Household Decision Making

We illustrate how capital and labor taxes affect the properties of the sharing rule in period 2. We derive our results under the following assumptions. First, as is customary in the literature (see for

\(^6\)Notice that the case where the participation constraints never bind corresponds to a full commitment allocation. Relative to this case, under limited commitment, the couple needs to partially give up risk sharing in order to satisfy the participation constraints.

\(^7\)In section 4 we formalize on the initial sharing rule assuming that the couple solves a Nash bargaining problem. For identical individuals the solution to this program would obviously be \( \lambda_1 = \frac{1}{2} \).
example Conesa et al. [2009]), we assume that individual utility is of the following form:

\[ u(c_t^g, l_t^g) = \begin{cases} 
   \frac{(c_t^g)^{\gamma}(l_t^g)^{1-\gamma}}{1-\gamma} & \text{for } \gamma > 1 \\
   \eta \log(c_t^g) + (1 - \eta) \log(l_t^g) & \text{for } \gamma = 1
\end{cases} \]

As we will later illustrate the exact specification of preferences (whether or not they are separable in consumption and leisure) has a significant impact on the effects of taxes in our model. We will therefore separately study the cases where \( \gamma = 1 \) and where \( \gamma > 1 \) in order to better highlight their properties. Second, for our baseline results we assume, as Mazzocco et al. [2007] do, that divorce leads to an equal division of assets and therefore set \( D_m(a_1) = D_f(a_1) = \frac{a_1}{2} \). As discussed previously, this assumption follows the empirical evidence. However, in section 3.2 we illustrate that our results go through under different specifications for divorce outcomes and divisions of assets.

### 3.1.1 Separable Preferences

Let us first assume that \( \gamma \) equals one. Assume that we are in period 2, and that given the sharing rule, the household decides how to split consumption and hours. Let \( A_c = (1 + r(1 - \tau_K))a_1 \) be the total financial income of the household brought forward from period one. It is trivial to show that the optimal consumption is \( c_2^m = \lambda_2 \eta(A_c + \sum_g w(1 - \tau_N)\epsilon_g) \) and \( c_2^f = (1 - \lambda_2)\eta(A_c + \sum_g w(1 - \tau_N)\epsilon_g) \).

Similarly, the optimal choice of leisure is given by: \( l_2^m = \lambda_2(1 - \eta)\frac{A_c + \sum_g w(1 - \tau_N)\epsilon_g}{\epsilon_m w(1 - \tau_N)} \) and \( l_2^f = (1 - \lambda_2)(1 - \eta)\frac{A_c + \sum_g w(1 - \tau_N)\epsilon_g}{\epsilon_f w(1 - \tau_N)} \).

**Participation Constraints.** To respect the participation constraint of each household member, the allocation rule \( \lambda_2 \) must satisfy the following conditions:

\[
\begin{align*}
\eta \log(\lambda_2 \eta(A_c + \sum_g w(1 - \tau_N)\epsilon_g)) + (1 - \eta) \log(\lambda_2 \eta(A_c + \sum_g w(1 - \tau_N)\epsilon_g)) + \xi & \geq 0 \\
\eta \log(\frac{A_c}{2} + w(1 - \tau_N)\epsilon_m) + (1 - \eta) \log(\frac{A_c}{2} + w(1 - \tau_N)\epsilon_m) & \geq 0
\end{align*}
\]

(6)

which guarantees that the male spouse is better off in the marriage than as a single and,

\[
\begin{align*}
\eta \log(1 - \lambda_2)\eta(A_c + \sum_g w(1 - \tau_N)\epsilon_g) + (1 - \eta) \log(1 - \lambda_2)\eta(A_c + \sum_g w(1 - \tau_N)\epsilon_g) + \xi & \geq 0 \\
\eta \log(\frac{A_c}{2} + w(1 - \tau_N)\epsilon_f) + (1 - \eta) \log(\frac{A_c}{2} + w(1 - \tau_N)\epsilon_f) & \geq 0
\end{align*}
\]

(7)

which represents the analogous condition for the female spouse. Solving equations (6) and (7) for \( \lambda_2 \) we get:

\[
\lambda_2 \in \begin{cases} 
   e^{-\xi} \frac{A_c}{A_c + \sum_g \epsilon_g w(1 - \tau_N)} & \text{for } \lambda_2^L \\
   1 - e^{\xi} \frac{A_c}{A_c + \sum_g \epsilon_g w(1 - \tau_N)} & \text{for } \lambda_2^U
\end{cases}
\]

(8)
Expression (8) defines an upper and a lower bound (\( \lambda^U_2 \) and \( \lambda^L_2 \) respectively) such that the participation constraints are satisfied. It also defines the updating rule for \( \lambda_2 \). For instance, given \( \epsilon_f \) and \( A_c \), there is a threshold \( \epsilon_m(A_c, \epsilon_f) \) such that if \( \epsilon_m < \epsilon_m(A_c, \epsilon_f) \), the contract updates \( \lambda_2 \) to be equal to the upper bound in (8) since in that case it holds that \( \lambda^U_2 < \frac{1}{2} \). Similarly, there is another threshold \( \epsilon_m(A_c, \epsilon_f) \) such that for an \( \epsilon_m \) above this threshold the new contract gives \( \lambda_2 \) equal to \( \lambda^L_2 \).

Note that when \( \xi = 0 \), \( \lambda^L_2 \) equals \( \lambda^U_2 \). In this case the intrahousehold allocation is such that in every state a household member’s share on total resources is essentially what they would get as a single. Being together is therefore no different than being single when \( \xi = 0 \).

**Labor Taxes.** Given the intrahousehold allocation, we can derive the effect of changes in the level of labor taxes \( \tau_N \) and capital taxes \( \tau_K \). First consider that labor taxes fall, i.e. \( (1-\tau_N) \) increases. We can write:

\[
\frac{d\lambda^L_2}{d(1-\tau_N)} = e^{-\xi}A_c \frac{\epsilon_m w - \frac{\sum g \epsilon_g}{2} (1-\tau_N) w^2}{(A_c + \sum g \epsilon_g (1-\tau_N) w^2)}
\]

\[
\frac{d\lambda^U_2}{d(1-\tau_N)} = e^{-\xi}A_c \frac{\sum g \epsilon_g w - \epsilon_f w}{(A_c + \sum g \epsilon_g (1-\tau_N) w^2)}
\]

Consider the case where \( \epsilon_m > \epsilon_m(A_c, \epsilon_f) \). In this case, it must be that \( \lambda_2 = \lambda^L_2 > \frac{1}{2} \), i.e. the male spouse’s weight needs to increase (the lower bound in (8) binds). The partial derivative (9) is positive, meaning that a reduction in the level of labor taxes increases \( \lambda^L_2 \) and, therefore, makes the change in \( \lambda_2 \) relative to \( \lambda_1 \) even greater. A fall in the tax rate reduces risk sharing within the household because in those states where the male labor income is high, the husband’s consumption share increases even more than if labor taxation is high. A similar argument can be made for the case where \( \epsilon_f >> \epsilon_m \). In that case the upper bound \( \lambda^U_2 \) will bind and the female spouse must be made better off. A reduction in \( \tau_N \) will decrease \( \lambda^U_2 \) hence making the fall in the male share larger.

We summarize the result in the following proposition:

**Proposition 1.** Assume that the household’s financial income is positive (\( A_c > 0 \)). A reduction in labor income taxes reduces insurance within the household under log-log (separable) preferences. The household sharing rule and thus the intrahousehold allocation become more responsive to changes in the idiosyncratic productivity of the male and the female spouse. 

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8It is worth emphasizing that the property that the couple can for any \( \epsilon_m, \epsilon_f \) pair replicate the ‘bachelor’ allocation is not a general feature of the dynamic model of intrahousehold bargaining. It holds in the two period model precisely because in \( t = 2 \) the household does not have to make an optimal savings decision. If we assume a third period, \( \epsilon_m > \epsilon_f \) and \( \xi = 0 \), then the male spouse would be better off outside the marriage since period 2 savings are split equally between the spouses in period 3. Such features are present in the dynamic model of Section 4 where we have to assume \( \xi > 0 \) in order to sustain an equilibrium without divorce.

9It is worth noting that general equilibrium effects that are left out from (9) and (10) typically operate in the opposite direction. To see this, note that in the short run, with the economy’s capital being fixed, a fall in labor taxes will affect wages and interest rates through hours worked. Since hours will increase in response to the fall in distortionary taxation, the wage rate \( w \) will decrease and the interest rate \( r \) will rise (under a Cobb-Douglas production technology). Consequently, the above expressions have to be multiplied by \( 1 + \frac{dw}{d(1-\tau_N)} (1-\tau_N) w \). Moreover, an additional term that reflects the effect of higher financial income on the constraint set has to be included. This term is given by \( \frac{dr(1-\tau_N)A_c}{d(1-\tau_N)} > 0 \) times the partial derivatives of \( \lambda^L_2 \) or \( \lambda^U_2 \) with respect to \( A_c \).

We can show that \( \frac{d\lambda^L_2}{dA_c} < 0 \) and \( \frac{d\lambda^U_2}{dA_c} > 0 \) is the case where \( \epsilon_m > \epsilon_f \) (male participation constraint is relevant). In words, when labor taxes fall, the rise in the interest rate will increase the importance of financial income to the household’s budget and relax the limited commitment constraints. This effect, however, will grow weaker as the
The intuition for this result is as follows: First, note that changes in labor income are the root of the limited commitment problem in the model. Therefore, a policy reform which lowers the tax rate, exacerbates the intra-household income inequality and exacerbates the limited commitment problem. This effect is captured by the fact that the partial derivative $\frac{d\lambda^L_2}{d(1-\tau_K)}$ is positive, at least insofar as financial income is positive. Second, to understand why wealth is important, notice that labor taxes, besides making the distribution of the net labor income more equitable in the household, they also impoverish the household by reducing its disposable income. When the household’s resources are fewer, the temptation to renege on the marital contract is stronger, and therefore commitment is less. This effect under log separable utility is relevant only insofar as $A_c < 0$. However, for the case where $\gamma > 1$ the effect will be present even if the household has savings rather than debt. We will return to this feature of the model in a later paragraph.

**Capital Taxes.** We derive the impact of wealth and capital taxation on the participation constraints. The relevant partial derivatives are those of $\lambda^L_2$ or $\lambda^U_2$ with respect to $1-\tau_K$, given by the following expressions:

\begin{align}
\frac{d\lambda^L_2}{d(1-\tau_K)} &= e^{-\xi}(1-\tau_N)\frac{\sum g \epsilon_g w - \epsilon_m w}{(A_c + \sum g \epsilon_g (1-\tau_N)w)^2} a_1 r \\
\frac{d\lambda^U_2}{d(1-\tau_K)} &= e^{-\xi}(1-\tau_N)\frac{\epsilon_f w - \frac{1}{2} \sum g \epsilon_g w}{(A_c + \sum g \epsilon_g (1-\tau_N)w)^2} a_1 r
\end{align}

Notice that if $\epsilon_m > \epsilon_f$ (male participation constraint may bind), (11) and (12) satisfy $\frac{d\lambda^L_2}{d(1-\tau_K)} < 0$ and $\frac{d\lambda^U_2}{d(1-\tau_K)} > 0$. They imply that a rise in asset income has a beneficial effect on household risk sharing. The next proposition summarizes the effect of lower capital taxation on household insurance:

**Proposition 2.** Lower capital taxes improve insurance under log-log preferences. The household sharing rule and thus intrahousehold allocation are less responsive to changes in labor income.

Several remarks are in order: First, notice that in a multi-period setting when capital taxes drop, more risk sharing may come from the higher net return on financial income, but also (in the longer term) because the household wants to save more in response to the higher return. It is also important to emphasize that the resulting change in intrahousehold insurance does not stem from the standard role of assets as a buffer against income shocks (e.g. precautionary savings), but rather is related to the division of resources in the household and the notion that wealth, in contrast to labor income, is a common resource for the household.  

---

10 The signs are opposite when $\epsilon_f > \epsilon_m$.

11 In the Appendix we investigate the impact of limited commitment on household savings using the two period model of this section.
Finally, equations (9) to (12) reveal that the effects of capital taxation, as well as the effects of labor taxes, depend on the wealth level of the household. The higher wealth is, the bigger the impact of the tax schedule on the intrahousehold allocation. This property implies that richer households are more likely to benefit from a change in policy which eliminates capital taxes. However, it is also worth noting that for these households the limited commitment friction is less severe, precisely because wealth is high. This property is key to understanding the modest effects from the reform that we uncover in our quantitative exercise in the next sections.

3.1.2 Non-Separable Preferences

We derive the impact of changes in taxation assuming $\gamma > 1$. Using simple algebra, we can write the participation constraints for male and female spouses as follows:

\begin{align}
(13) \quad & \left( \frac{A_c + \sum_g w(1 - \tau_N)\epsilon_g}{(1 + f(\lambda_2, \epsilon))((w(1 - \tau_N)\epsilon_m)^{1-\gamma})} \right)^{1-\gamma} \frac{\chi}{1 - \gamma} + \xi \geq \left( \frac{\frac{A_c + w(1 - \tau_N)\epsilon_m}{w(1 - \tau_N)\epsilon_m}^{1-\gamma}}{(1 + f(\lambda_2, \epsilon))((w(1 - \tau_N)\epsilon_m)^{1-\gamma})} \right)^{1-\gamma} \frac{\chi}{1 - \gamma} \\
(14) \quad & \left( \frac{f(\lambda_2, \epsilon)}{(1 + f(\lambda_2, \epsilon))} \frac{A_c + \sum_g w(1 - \tau_N)\epsilon_g}{(w(1 - \tau_N)\epsilon_f)^{1-\gamma}} \right)^{1-\gamma} \frac{\chi}{1 - \gamma} + \xi \geq \left( \frac{\frac{A_c + w(1 - \tau_N)\epsilon_f}{w(1 - \tau_N)\epsilon_f}^{1-\gamma}}{(1 + f(\lambda_2, \epsilon))((w(1 - \tau_N)\epsilon_f)^{1-\gamma})} \right)^{1-\gamma} \frac{\chi}{1 - \gamma}
\end{align}

where $\chi = (\eta^\gamma(1 - \eta)^{1-\gamma})^{1-\gamma}$ and $f(\lambda_2, \epsilon) = \left( \frac{\lambda}{\lambda_2} \frac{\epsilon^\gamma}{\epsilon_m} \right)^{(1-\eta)(1-\gamma)} - 1/\gamma$. We show in the Appendix that the sign of the effect of a rise in $1 - \tau_N$ on $\lambda^*_2$ is inversely related to the sign of the following expression:

\begin{align}
(15) \quad & (1 - \gamma) \left[ \tilde{\zeta} \left( \frac{A_c(1 - \eta)}{1 - \tau_N} - \eta \sum_g w\epsilon_g \right) \frac{\kappa_1(A_c, \epsilon) + (A_c w(\epsilon_m - \frac{\sum_g \epsilon_g}{2})) \kappa_2(A_c, \epsilon)}{A_c} \right]
\end{align}

where $\kappa_1 = \frac{(w(1 - \tau_N)\epsilon_m)^{(1-\eta)(1-\gamma)}}{(A_c + \sum_g w(1 - \tau_N)\epsilon_g)^{1-\gamma}} > 0$, $\kappa_2 = \frac{(\frac{A_c + w(1 - \tau_N)\epsilon_m}{(A_c + \sum_g w(1 - \tau_N)\epsilon_g)^{1-\gamma}})^{-\gamma}}{w(1 - \tau_N)\epsilon_f} > 0$ and $\tilde{\zeta} = \frac{-\xi(1-\gamma)}{\eta^\gamma(1-\eta)^{1-\gamma}} > 0$.

In (15), if $A_c = 0$, only the leading term is different from zero and in fact it is positive. In this case, we can show that an increase in $1 - \tau_N$ will increase intrahousehold risk sharing against uncertain labor income, or to put it differently, it will reduce the response of the sharing rule to variations in income. When $A_c > 0$, the second term is added and also the first term in (15) eventually switches sign. When the overall partial derivative is negative, the fall in labor taxation exacerbates inequality within the family and reduces welfare by reducing insurance. This is the result we established under log-log separable utility. What non-separability brings to the equation is the leading term in (15) which yields the non-monotonicity that makes the effect of changes in labor income taxation ambiguous.

To understand what this term captures, note that, as discussed previously, a rise in the tax rate reduces inequality in terms of labor incomes but it also impoverishes the household. When utility is curved, the reduction in household income translates into an increase in the marginal utility which may tighten the participation constraints. In the case of log utility, this effect was balanced by the larger equity in household resources; it dominated only when the household had negative savings. But under $\gamma > 1$ the income effect may dominate even when household financial income is positive. In the Appendix, we show that a similar result applies to the upper bound $\lambda^*_2$. We also establish that the effect of lower capital taxation is unambiguous; it always improves the household’s insurance possibilities.
Proposition 3. Assume \( A_c > 0 \) and \( \gamma > 1 \). Lowering labor income taxes reduces intrahousehold inequality when household wealth is low. In contrast, when the household is wealthy, reducing labor taxation has a detrimental effect on intrahousehold insurance. Lower capital taxes always improve the household’s insurance possibilities.

Proof: See Appendix.

In our quantitative analysis in Section 5, following the rest of the literature on optimal taxation, we deal only with the case of non-separable utility. However, we think that it is important to disentangle the channels through which preferences affect our results and therefore we also consider here a commonly used specification of separable preferences. Moreover, note that since the change in policy considered in Section 5 is one which eliminates capital taxation and raises labor income taxes, our results under non-separable utility illustrate that for some households, the limited commitment problem may even be more severe after the reform. This could be the case for young households, which are typically wealth poor. We will investigate in Section 5 whether this holds.

3.2 Implications and Alternative Approaches

Equations (13) and (14) in the previous paragraph defined the updating rule of the household weight \( \lambda \). According to the household contract, the weight jumps if and only if there is a violation of the participation constraint, and if not, it stays constant over time. Rewriting equation (13) we can express the updating rule when the husband’s participation constraint binds as follows:

\[
\frac{\lambda_2}{1 - \lambda_2} = \frac{\epsilon_m}{\epsilon_f} \left[ (1 - \eta)(1 - \gamma) \right] \frac{\xi e^{(1 - \eta)(1 - \gamma)}(1 - \gamma)}{(A_c + \sum g \epsilon_g w(1 - \tau_N))^{1 - \gamma}} + \frac{A_c + \epsilon_m w(1 - \tau_N)}{A_c + \sum g \epsilon_g w(1 - \tau_N)} \gamma \right]^{-1} - 1)^{-\gamma}
\]

We think that it is important to generalize our findings, by briefly explaining what (16) would look like if we included in our model the following features: 1. Preference heterogeneity, 2. Nash Bargaining each period and 3. Different costs of divorce and a different division of the household’s wealth in the event of divorce.

To address preference heterogeneity, we keep our parameterization of the individual utility function as non-separable but assume that male and female spouses value consumption and leisure differently through the parameters \( \eta_m \neq \eta_f \). Note that given our specification of preferences, differences in \( \eta_m \) and \( \eta_f \) also translate into differences in the relative risk aversion coefficient of the spouses.\(^{12}\)

It can be shown that the updating rule takes the following form:

\[
\frac{\lambda_2}{1 - \lambda_2} = \kappa_3 \left( \frac{(1 - \eta_m)(1 - \gamma)}{\xi e^{(1 - \eta_f)(1 - \gamma)}} \right) \left[ -\frac{\xi e^{(1 - \eta_m)(1 - \gamma)}(1 - \gamma)}{(A_c + \sum g \epsilon_g w(1 - \tau_N))^{1 - \gamma}} + \frac{A_c + \epsilon_m w(1 - \tau_N)}{A_c + \sum g \epsilon_g w(1 - \tau_N)} \gamma \right]^{-1} - 1 \right]^{-\gamma}
\]

where \( \kappa_3 = \left( \frac{2}{\eta_m - \eta_f} \right)^{1 - \gamma} \left( \frac{\eta_m}{1 - \eta_m} \right)^{(1 - \eta_f)(1 - \gamma)} \left( \frac{\eta_f}{1 - \eta_f} \right)^{(1 - \eta_f)(1 - \gamma)} \). Notice that (17) is different from (16) only in the leading term. In both equations the leading terms are positive and the crucial expression which determines sign of the effect of wealth and the tax rates is the bracketed subsequent term. Therefore under preference heterogeneity wealth continues to have the same impact on household commitment and the implications of changes in capital and labor taxes on commitment are essentially the same.

\(^{12}\) It is not possible to derive closed form solutions when \( \gamma_m \neq \gamma_f \). We have however solved this case numerically and found that our results generalize.
Under Nash Bargaining the household contract no longer has the property that it keeps individual weights constant over a region of the state space where the participation constraints do not bind. In the Nash bargaining equilibrium the sharing rule is rebargained each period, and \( \lambda_2 \) is a function of assets, \( \epsilon_m \) and \( \epsilon_f \) and solves the following equation:

\[
\lambda_2(a, \epsilon) \in \text{arg max}_{\lambda_2} \left( \left( \sum \epsilon_g m \right) + \sum \epsilon_g (1 - \tau_N) \epsilon_g \right)^{1-\gamma} \frac{\chi}{1 - \gamma} + \xi - \left( \frac{\lambda_2 + \epsilon_m (1 - \tau_N) \epsilon_m}{(1 - \tau_N) \epsilon_m} \right)^{1-\gamma} \frac{\chi}{1 - \gamma}
\]

From (18) we can derive the impact of wealth on the equilibrium allocation that solves the Nash bargaining program. For the sake of brevity we leave it to the Appendix to show that as wealth increases the surplus for the household increases and the equilibrium weight gets closer to 1/2. Shocks to individual income have a smaller impact on the weight the higher the stock of wealth. The analysis of the previous section, which derives the impact of the tax schedule on the household sharing rule, is therefore applicable to this model.\(^{13}\)

Finally, assume that instead of an equal division of assets in the case of divorce we have a rule of the form \( A_c \geq D_m(A_c) + D_f(A_c) \). As in Regalia and Ríos-Rull [2001] this specification allows for divorce costs to be proportional to the wealth stock but also for male and female shares to be different. Under this rule we can establish that the sign of the effect of wealth on rebargaining, when the male spouse’s participation constraint binds, is determined by:

\[
\frac{(\gamma - 1)2\xi \epsilon_m (1-\eta) (1-\gamma)}{\chi(A_c + \sum \epsilon_g) 1-\gamma} + (1 - \gamma) \kappa_4 \left[ D_m' A_c - D_m(A_c) + w(1 - \tau_N) (D_m' \sum \epsilon_g - \epsilon_m) \right]
\]

where \( \kappa_4 = \frac{(D_m(A_c) + \epsilon_m (1 - \tau_N))^{-\gamma}}{(\sum \epsilon_g w(1 - \tau_N) + A_c)^{-\gamma}} \). The first term captures more commitment when wealth increases. The second term has the same impact if positive. To simplify, let \( \theta_m \) be the male spouses share on wealth in the event of divorce. Then the second term in (19) becomes \((1 - \gamma) \kappa_4 \left[ w(1 - \tau_N)(\theta_m \sum \epsilon_g - \epsilon_m) \right]\). This is positive surely over the relevant region if \( \theta_m \leq \frac{1}{2} \), because (19) is pertinent only when \( \epsilon_m > \epsilon_f \). Moreover, even if this condition is violated, i.e. when \( \theta_m > \frac{\epsilon_m}{\sum \epsilon_g} \), \( \theta_m \) still needs to be large enough to compensate for the first term in (19).

As discussed previously, the relevant empirical evidence suggests that US courts in principle on average split equally the marital property (assets). In this respect, the weight \( \theta_m \) could be large, but only as a realization of a stochastic \( \theta_m \) that the couple draws after the divorce is final. Notice that if we had made \( \theta_m \) stochastic, then for risk averse individuals divorcing would possibly be even less attractive the higher wealth is, because the variability in post divorce consumption would be higher. We conclude this paragraph by noting that under reasonable alternative assumptions our model’s

\(^{13}\)Notice that given that \( \lambda_2(a, \epsilon) \) is a function of the state vector under Nash bargaining, the equilibrium under Nash bargaining is a Markov perfect equilibrium whereby wealth and current productivity are sufficient to summarize the household’s contract. In contrast, under our benchmark contract the entire history of shocks matters for the allocation (even beyond the current stock of wealth and productivity). Ours is a model where the weight is constant over a part of the state space, and in this region perfect risk sharing applies. Nash bargaining is therefore less efficient because it leads to more frequent renegotiations over the intrahousehold allocation.
implications continue to hold.  

4 Quantitative Model

This section presents our quantitative life cycle model. We consider an economy populated by a continuum of individuals, equally many males and females. Gender is indexed by \( g \in \{m, f\} \) and age by \( j \in \{1, 2, ..., J\} \). Individuals survive from age \( j \) to \( j + 1 \) with probability \( \psi_j \). At each date a new cohort of individuals enters the economy; we assume that the population grows exogenously at rate \( \theta \).

The life cycle of individuals comprises of the following three stages: Marriage (matching), work and retirement. Since our model does not endogenize family formation, we simplify the first stage by letting matching take place in a pre-labor-market period of life labeled age zero. A fraction \( \mu \) of individuals will find partners at this stage and form households as couples, and the remaining agents will remain bachelors. Marital status does not change over time. After date zero individuals work for \( j_R - 1 \) periods –conditional on survival– and then retire at date \( j_R \). At age \( J \), they die with probability one.

4.1 Endowments

Agents in the economy differ in terms of their labor productivity along three dimensions: a deterministic (life cycle) component \( L_g(j) \), a fixed effect \( \alpha_g \) and an idiosyncratic labor productivity shock \( \epsilon_g \). When entering the labor market, each agent draws a realization of \( \alpha_g \), the value of which remains constant throughout their working life. We assume that there are \( N \) possible realizations \( \{\alpha_{1,g}, \alpha_{2,g}, ..., \alpha_{N,g}\} \) for each gender. The assignment of a realization is made according to some probabilities \( p^S_g \) when the agent is single, and according to probabilities \( p^M \) when the individual is married. Note that \( p^M \) is the joint distribution of the two spouses across all possible values of \( \alpha_m \) and \( \alpha_f \). Idiosyncratic productivity \( \epsilon_g \) changes stochastically over time according to a first order Markov process. We let \( \pi_g(\epsilon'_{g}|\epsilon_g) \) be the conditional pdf for this process. The analogous object for couples is denoted by \( \pi(\epsilon'_{g} | \epsilon_{g}) \), where \( \epsilon \) is the vector of productivities of spouses.

\[\text{\footnotesize \textsuperscript{14}}\text{For the sake of completeness we briefly comment on how our model’s implications would be affected if we allowed divorce settlements to depend on the income of the male and the female spouses such as in the case of alimony. First, note that alimony payments are more likely to concern a permanent component of individual productivity rather than a transitory shock such as the } \epsilon \text{ endowment considered in this section. In our quantitative model we allow for a fixed effect component to labor income and a life cycle component of male and female productivity implicit to which is the gender pay gap. We argue that by and large these predictable components of individual productivity are factored in the initial allocation in the first period, and do not lead to renegotiations of the marital contract. Household rebargaining (or its most part) stems from the time varying transitory shocks which are similar to the shocks considered in this section.}

Second, note that when wealth is a state variable, as it is in our model, past realizations of temporary shocks do affect the outside options of individuals through wealth accumulation. Families that have experienced a stream of good shocks in productivity, are likely to have accumulated more wealth. Given our formalization of how wealth is divided in the event of separation it is therefore obvious that in the multiperiod model of the next section, outside options, through wealth, are influenced by the history of individual labor income and are thus (partially) contingent on the realization of shocks. If one of the spouses experiences a stream of high productivity shocks, he (she) contributes more to wealth accumulation and in the event of a separation, the division of common marital property favors the other spouse. Notice also that for couples with enough wealth, modelling such transfers through the division of wealth or modelling them as per period lump sum payments is essentially the same.
4.2 Markets and Technology

The production technology is Cobb-Douglas:

\begin{equation}
Y = K^\alpha (AN)^{1-\alpha}
\end{equation}

where \( K \) denotes the economy’s aggregate capital stock, and \( N \) is the aggregate labor input, and \( A \) is the level of labor-augmenting technology. The resource constraint is given by \( K' = (1 - \delta)K + Y - G - C \), where by convention, primes denote the next model period. \( C \) is aggregate consumption in the economy, \( G \) is government spending and \( \delta \) is the depreciation rate of the aggregate capital stock. Factor prices are determined in competitive markets. Wages, measured in efficiency units, are equal to the marginal product of labor, and the return to capital is its marginal product net of depreciation. We denote these objects by \( w \) and \( r \) respectively.

Financial markets are incomplete. There are no state contingent securities. By trading claims on the aggregate capital stock, agents can self insure. In keeping with the literature, we assume that these trades are subject to an ad hoc borrowing constraint \( \bar{a} \). The value for the constraint is set to zero, such that our economy rules out borrowing altogether. Moreover, there are no annuity markets and households leave accidental bequests which we denote by \( B \). Bequests are distributed uniformly across individuals in the economy.

4.3 Government

The government engages in two activities. First, it levies taxes on consumption \( \tau_C \), on financial income \( \tau_K \), and on labor income \( \tau_N \) to finance a level of expenditures \( G \). We rule out government debt so that the government runs a balanced budget each period. Second, it runs a Pay-as-you-Go social security system, which is financed through a proportional tax on the earnings of the working population. We denote by \( \tau_{SS} \) the social security tax, and by \( SS(g, \alpha_g) \) the transfer that a retired individual receives from the government. Notice that transfers depend on gender \( g \), and on the wage fixed effect \( \alpha_g \). Our aim with this formulation is to capture the current US social security system in a parsimonious way. \( SS(g, \alpha_g) \) depends also on gender because life cycle productivity \( L_g(j) \) differs across men and women in the economy.

4.4 Value Functions

Bachelor Households. We first consider the program of a bachelor of gender \( g \) and age \( j \). We let \( S_g(a, X, j) \) be the lifetime utility for this agent when her (his) stock of wealth is \( a \), her permanent productivity is \( \alpha_g \), her idiosyncratic time varying productivity is \( \epsilon_g \). To save on notation, we summarize the fixed effect and the time varying component of productivity in a vector \( X \). This agent must choose consumption \( c \) and hours worked \( n \) (if not retired, i.e. \( j < j_R \)) to maximize her utility
subject to the budget and the borrowing constraints. She solves the following functional equation:

\[
S_g(a, X, j) = \max_{a' \geq a, l} u(c, l) + \beta \psi_j \int S_g(a', X', j + 1) d\pi_g(X'|X)
\]

Subject to:

\[
l + n \leq 1
\]

\[
a' + (1 + \tau_C)c = (a + B)(1 + r(1 - \tau_K)) + w\epsilon_g \alpha_{i,g} L_g(j)(1 - \tau_N - \tau_{SS})n \quad \text{if } j < j_R
\]

\[
a' + (1 + \tau_C)c = (a + B)(1 + r(1 - \tau_K)) + SS(g, \alpha_g) \quad \text{if } j \geq j_R
\]

**Couples.** In this paragraph, we describe the program of the couple. As in section 3, we model decision making within the household as a contract under limited commitment following the work of Ligon et al. [2000], Mazzocco et al. [2007], and Voena [2012]. We denote the male share with \( \lambda \) and with \( 1 - \lambda \) the female spouse’s share. As previously, given the household’s contract these shares will change over time to satisfy participation. However, in the model of this section, rather than imposing the initial value of \( \lambda \), we assume that it solves a Nash bargaining program at the matching stage of the life cycle.

Let \( M(a, X, \lambda, j) \) be the value function of a household of age \( j \), where \( X \) summarizes the productive endowments of its members.\(^{15}\) As before \( a \) is the level of wealth, and \( \xi \) is the (constant) benefit that accrues to each spouse in the marriage. The program of the couple can be written as:

\[
M(a, X, \lambda, j) = \max \lambda u(c_m, l_m) + (1 - \lambda) u(c_f, l_f) + \xi
\]

Subject to:

\[
l^g + n^g \leq 1 \quad \text{for } g \in \{m, f\}
\]

\[
a' + (1 + \tau_C)(c^m + c^f) = (a + 2B)(1 + r(1 - \tau_K)) + w\left(\sum_g L_g(j)\alpha_{i,g} n^g(1 - \tau_N - \tau_{SS})\right) \quad \text{if } j < j_R
\]

\[
a' + (1 + \tau_C)(c^m + c^f) = (a + 2B)(1 + r(1 - \tau_K)) + \sum_g SS(g, \alpha_{i,g}) \quad \text{if } j \geq j_R
\]

Notice that in (22), the couple draws a new value \( \lambda' \) in the next period. As discussed previously, this updating occurs if there is a violation of participation. If, for example, under a new realization of the state vector \( X' \) the husband is better off as a single than under the contract \( \lambda \), his share in household resources must increase to reflect his improved bargaining position. Formally, the updating rule for \( \lambda \) (which generalizes the analogous object in section 3) is:

\[
\lambda' \in \arg \min_{\lambda^*} |\lambda^* - \lambda| \quad \text{such that}
\]

\[
V_g(a', X', \lambda^*, j + 1) \geq S_g(\frac{a'}{2}, X'_g, j + 1)
\]

where \( X_g \) is a vector formed by elements of \( X \) that are relevant to household member \( g \) if he or she

\(^{15}\)In our economy households are formed by agents of the same age cohort. This assumption is reasonable given that one period in the model is five years. Moreover, we impose that household members die together.
were to be single.

The value of $\lambda$ is initiated at the matching stage of the life cycle, as a solution to the following Nash bargaining problem:

$$
\lambda_1 \in \arg \max_{\lambda} \left[ V_m(a, X, \lambda, 1) - S_m \left( \frac{a}{2}, X, \lambda, 1 \right) + \xi_m \right] - 
\left[ V_f(a, X, \lambda, 1) - S_f \left( \frac{a}{2}, X, \lambda, 1 \right) + \xi_f \right]
$$

One final comment is in order: Notice that the Nash sharing rule determines the initial allocation under the influence of two additional gender specific utility gains $\xi_g$ for $g \in \{m, f\}$. These gains at the matching stage determine the magnitude of income transfers from one spouse to the other. For example, if $\xi_m > \xi_f$ then the household contract will give an initial allocation with large transfers from the male to the female spouse, thus leading to a big inequality in hours within the family. We will choose values for $\xi_m$ and $\xi_f$ to target the division of hours as in the US data.

We characterize the competitive equilibrium in section A.4 in the Appendix.

4.5 Calibration and Model Evaluation

Preferences and Demographics. The demographic parameters have been set so that the model has a stationary demographic structure that matches the age distribution in the US economy. We assume that individuals are born at age 25 and live at most until age 95. Retirement is at age 65. The survival probabilities are taken from Arias [2010], based on the US National Vital Statistics Reports and correspond to probabilities concerning the entire population (pooling men and women). The model period is set to five years. This means that there are fifteen periods, and the retirement age is $j_R = 10$. Although we make this assumption for computational reasons, in what follows we report annual values for the parameters.

We set the fraction of households that are married ($\mu$) equal to 52%, which is the corresponding statistic in the PSID data over all age groups. With this choice roughly 69% of all individuals in our economy are married. Population growth is assumed to be $\theta = 0.012$. We calibrate the preference parameters as follows: first, we follow Conesa et al. [2009] and Fuster et al. [2008], and set $\gamma = 4$. $^{16}$ We also choose a value of $\eta = 0.41$ so that our model produces, in the steady state, average hours worked of one third. With these numbers the intertemporal elasticity of substitution, $(1 - \eta(1 - \gamma))^{-1}$, is equal to 0.4484.

For married couples we have to determine the utility gains $\xi_m$ and $\xi_f$ at the Nash bargaining stage, and the flow gain $\xi$ that couples enjoy at each period. As discussed earlier, these parameters govern the following two aspects of the intrahousehold allocation: First, $\xi_m$ and $\xi_f$ determine the transfers from one spouse to the other, and along with differences in the age productivity profiles $L_j(g)$, they determine inequality in hours within the household. We pick numbers for these parameters to match average hours worked for married men and women as in the US data; according to the PSID married males worked 2104 hours in 2006, married females 1420 hours. $^{17}$ We map these numbers into model

$^{16}$ This choice is consistent with the empirical evidence of Attanasio and Weber [1995] and Meghir and Weber [1996].

$^{17}$ Average hours are reported in the PSID 2007 survey and correspond to the previous year work time.
Second, as was made evident from our previous analysis, $\xi$ determines the ability of the household to commit to a sharing rule $\lambda$. The smaller $\xi$ is, the more the household members will be tempted to renege on this rule and the more frequent rebargaining will be. Our strategy to calibrate the value of $\xi$ is the following: We choose a value that is as low as possible so that we maximize the frequency of rebargaining in the model but also so that divorce never occurs in equilibrium. In this sense, we exacerbate the limited commitment friction so as to give to our model the best chance to yield a large gain in terms of commitment from the shift in the tax schedule.

**Technology and Endowments.** The technology parameters are chosen as follows: We allow $A_t$ to grow at a rate equal to 1.4% per year. Moreover, we set the capital share in value added $\alpha$ equal to 0.36 and we choose the depreciation rate of capital $\delta$ to match an investment to output ratio of 21% in the steady state. This gives us an annual value for this parameter of 5.26%. The subjective discount factor $\beta$ is calibrated so that the economy in the steady state produces the capital output ratio of 2.7. This procedure yields a value of 0.998.

Individual wages in the model are the product of three components; the gender specific life cycle profile, the fixed effect, and the temporary idiosyncratic productivity shock. Following the bulk of the literature, we take the life cycle profiles $L_g(j)$ from Hansen [1993]. Moreover, our principle to calibrate the fixed effect component and the distribution of the idiosyncratic shocks $\epsilon$ is to reproduce the life cycle pattern of the cross-sectional variance of family earnings as is documented in Storesletten et al. [2004]. For the fixed effect, we choose two values $\alpha_1, \alpha_2$ which we assume are common across gender and marital status. We follow Conesa and Krueger [2006] and assume that $\alpha_1 = e^{-\sigma_\alpha}$ and $\alpha_2 = e^{\alpha_0}$ where $\sigma_\alpha$ governs the variance of the process. For bachelor agents we calibrate the fractions $p^q_g$ to 1/2. For couples, we calibrate the joint probabilities $p^M$ so that our economy reproduces the degree of marital sorting in earnings ability that has been documented in the US data; for instance Hyslop [2001] estimates a 0.5 correlation of fixed effects within the family in his PSID sample. To match this correlation, we set $p^M = 0.375$ for households where $\alpha_m = \alpha_f$ (i.e. spouses have the same fixed effect) and $p^M = 0.125$ otherwise. We choose $\sigma_\alpha$ to reproduce the cross-sectional variance of household income at age 25 reported in Storesletten et al. [2004]. We also choose the persistence and the variance of the innovation to $\epsilon$ to make the model produce a linear rise in the

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18 Our results are not sensitive to the calibration of the initial bargaining position. It is worth noting however that $\xi_m$ and $\xi_f$ are not the only parameters that affect the initial sharing rule, but also the constant gain $\xi$. The higher is $\xi$ and the closer is the model to the full commitment allocation (see the previous discussion), the closer are the initial shares to $\frac{1}{2}$ by the properties of Nash bargaining. Our approach is to utilize $\xi$ to maximize rebargaining in the family (so a low value is targeted) and use $\xi_m$ and $\xi_f$ in order to target the division of hours as in the data. This gives us $\xi_m > 0$ and $\xi_f < 0$; nevertheless the female (initial) share $1 - \lambda$ is underestimated under the limited commitment contract and is roughly 0.45 on average. To put this differently, absent intrahousehold transfers the share would have been even lower and female hours even higher. If on the other hand the household contract was complete, transfers from males to females would be higher and married females would work too little relative to the targets in the data.

19 Note that a model which contains home hours as well as market hours would possibly enable us to set $\xi_m$ and $\xi_f$ equal to zero, because overall hours in the household would reflect specialization in home and market production rather than initial differences in the sharing rule. This is an important extension that we leave to future work.

20 The appropriate value of $\xi$ is determined so that divorce never occurs in the steady state, but also after the policy change (in the new steady state and in the transition). In particular, we find that divorce given $\xi$, is more frequent with the new policy, leading us to choose a higher $\xi$ than what is required to avoid divorces in the initial steady state. We will comment further on this implication of the model in a subsequent section.

21 In order to represent our economy in the computer, we have to make the standard normalizations as in Aiyagari and McGrattan [1998].

22 Our calibration targets are taken from Fuster et al. [2008].
cross-sectional variance with age and a value of 0.9 at age 65. Finally, we allow for shocks to ε to be contemporaneously correlated within the family and set the correlation equal to 0.15 (see Hyslop [2001]).

**Government.** In order to parameterize the steady state tax code, we proceed as follows. The level of expenditures $G$ is chosen so that on the balanced growth path, the government consumes 21% of output. This spending is financed by the tax levies on consumption, capital and labor income. We follow Fuster et al. [2008] and fix the consumption tax $\tau_C$ to 0.05, and we set the steady capital income tax $\tau_K$ to 0.35. The value of $\tau_N$ is chosen so that the government runs a balanced budget. In the initial steady state the model gives us a value of roughly 15% for this parameter. In our numerical experiment, we eliminate capital income taxation and adjust labor income taxes, while holding the level of expenditures constant to their steady state value.

Our principle to calibrate the social security benefit system is the following: First, as explained previously, we consider the individual as the unit to which benefits are distributed and not the household. Second, we try to capture in a parsimonious way, with the functional $SS(g,a_g)$, the fact that social security in the US contains a redistributive component. For instance, in 2004 individuals received 90% of the first 7,300 of their total social security entitlement, 32% for earnings between 7,300 and 44,000, and 15% for earnings above 44,000. We calibrate $SS(g,a_g)$ so that the model economy gives roughly the same redistribution of income in retirement in terms of median lifetime earnings as in the US economy. To give an idea of the numbers, we calculate that men of the highest earning ability (fixed effect) get roughly 1.5 times as much as men of the lowest ability, whereas their lifetime earnings are twice as high. Furthermore, women with the lowest ability receive in benefits roughly 67% of what poor men receive, and women of high ability get slightly (3%) more than poor men. We fix the social security tax rate at 12.5% and solve for the average level of benefits. In the computational experiment, we keep the tax rate constant across environments and vary the level of benefits.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$ fraction married households</td>
<td>0.52</td>
</tr>
<tr>
<td>$\theta$ annual population growth</td>
<td>1.2%</td>
</tr>
<tr>
<td>$\dot{A}$ annual productivity growth</td>
<td>1.4%</td>
</tr>
<tr>
<td>$\alpha$ capital share</td>
<td>0.36</td>
</tr>
<tr>
<td>$\delta$ annual depreciation rate</td>
<td>5.26%</td>
</tr>
<tr>
<td>$\beta$ discount factor</td>
<td>0.998</td>
</tr>
<tr>
<td>$\gamma$ preference parameter</td>
<td>4</td>
</tr>
<tr>
<td>$\eta$ preference parameter</td>
<td>0.41</td>
</tr>
<tr>
<td>$\tau_C$ consumption tax</td>
<td>5%</td>
</tr>
<tr>
<td>$\tau_K$ capital income tax</td>
<td>35%</td>
</tr>
<tr>
<td>$\tau_{SS}$ social security tax</td>
<td>12.5%</td>
</tr>
<tr>
<td>$\tau_N$ labor income tax</td>
<td>14.9%</td>
</tr>
</tbody>
</table>

Note: For growth rates ($\dot{A}$, $\theta$, $\delta$) the reported values are annual, but we solve on 5-yearly frequency.

23We discretize $\epsilon$ as a Markov process using standard techniques.
4.5.1 Model Performance

In table 2 we summarize relevant cross-sectional observations in the model and in the data. Because our focus is on the behavior of couples we report our model’s performance in matching a set of cross-sectional moments that refer to married households. In the first row we look at between married household wealth inequality. According to the results in the table the model produces a wealth Gini coefficient of 0.61 whereas in the data the analogous quantity is in the order of 0.64. Note however, that given the structure of the PSID, we cannot disentangle which part of a household’s net worth is accumulated in the marriage, and which part was brought into the marriage by the spouses. On the other hand, in the model, all wealth is common marital property. In this respect the data Gini may slightly overestimate the between married household inequality in wealth.

In the second row of the table we report the Gini index for between household income inequality. This statistic reflects the sum of male and female labor income (total family labor income) and therefore refers to the income distribution for US couples. The model matches the data counterpart very accurately, generating a Gini coefficient of 0.35 vs. 0.33 in the data. Rows 3 to 5 summarize inequality in income and wages within the household. The third row reports the average fraction of male to total household labor income, the fourth row the mean difference between the log male income and the log of female income, and the fifth column reports the analogous difference for wages. In all of the statistics the model matches the US data very well. In particular, on average, married men earn 64% of total family income in the US; the model produces a value of 68%. The average of the difference of the logs of income (wages) is 0.68 (0.68) in the model, and it is 0.64 (0.65) in the data.

As discussed earlier we have targeted average hours worked for married men and married women as in the US data. In the last two rows of the table we investigate how well the model performs in matching male and female participation to the labor market. Note that in spite of the fact that there is no explicit extensive margin in the model, individuals may nonetheless choose zero hours, in which case the optimal allocation is at a corner solution. In the data columns we report the fraction of working age households which, in a five year interval in the PSID, have positive hours. This fraction is 89.4% in the data for married women and it is 97% for married men. The model generates numbers for these statistics of 89.6% and 98.7% respectively. Therefore, on average it can match very accurately non-participation.

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It is well known that overall between household wealth inequality in the US is considerably higher. For instance, the wealth Gini for all household types (singles and couples) is in the order to 0.75 in the data. Evidently an important variable in explaining wealth inequality is marital status (as documented by Guner and Knowles [2007]).

Note that even though we have assumed that the male and the female preferences are the same, the model can generate differences in labor supply elasticities by gender following an argument similar to Alesina et al. [2011]. To see this consider the following formula for the elasticity derived from a log linear approximation of the labor supply optimality condition: 

\[ e_g = \frac{\ell_g}{1 - \ell_g} \]

where \( \ell_g \) denotes the average leisure of gender \( g \) in the model. According to this expression the spouse which works less hours, has a more responsive labor supply to changes in taxes and productivity. Though the argument is derived locally, it is relevant for our model: to evaluate this we simulate a panel of households and estimate the first order condition for hours; our estimates show that the female elasticity in the model is 1.67 and the analogous quantity for married men is 1.3.
Table 2: Cross-Sectional Moments: Model and Data

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gini Wealth</td>
<td>0.61</td>
<td>0.64</td>
</tr>
<tr>
<td>Gini Income</td>
<td>0.35</td>
<td>0.33</td>
</tr>
<tr>
<td>Male to Total Household Income</td>
<td>0.68</td>
<td>0.64</td>
</tr>
<tr>
<td>Log Income Ratio</td>
<td>0.68</td>
<td>0.65</td>
</tr>
<tr>
<td>Log Wage Ratio</td>
<td>0.31</td>
<td>0.33</td>
</tr>
<tr>
<td>Female Participation</td>
<td>89.6%</td>
<td>89.4%</td>
</tr>
<tr>
<td>Male Participation</td>
<td>98.7%</td>
<td>97.0%</td>
</tr>
</tbody>
</table>

5 Results

This section contains our main quantitative results. We evaluate how a reform which sets capital taxation permanently equal to zero and raises labor taxes affects the economy, in terms of aggregates, welfare and the intrahousehold allocation. In order to fully investigate the impact of this change in the tax schedule, we consider both the long run and the short run effects, meaning both the behavior of the economy in the final steady state as well as in the first period of the transition.

5.1 Long run and Short run Effects on Aggregates

In figure 1 we plot the response of aggregate capital (top left), aggregate output (top right), benefits (bottom left) and returns (bottom right). Period zero stands for the old steady state, before the reform. In period one there is an unanticipated permanent shift in the tax schedule. The figure shows the path of the variables from the initial period until the new steady state is reached, after 30 model periods.

Notice that aggregate capital is predetermined and therefore does not change when the reform takes place. However, when capital taxation is abolished individuals are more willing to save due to the rise of the after tax return on savings. We represent the net return with the dashed line in the bottom right panel. It increases by roughly 41% on impact, and even in the long run is considerably higher than in the initial steady state. The solid line in the plot, which represents the gross return, indicates that the build up of capital lowers the gross return to savings by 11% in the final steady state.

Notice that when the reform takes place there is a considerable drop in aggregate output deriving from the response of hours to the change in the tax schedule. In table 3 (rows 2 and 3) we report the behavior of hours and labor taxes. Aggregate hours fall by roughly 8.5% initially and labor taxes increase by 201% in period 1, and by 193% in the final steady state. The reason for this pattern is that higher capital eventually increases wages and individual labor income which, after the reform, is the tax base. Therefore, in the long run a lower labor tax is needed to balance the government’s budget than in the first period of the transition. Moreover, due to the joint behavior of capital and hours, output falls by 5.25% initially and then rises until the new steady state. After thirty periods aggregate output is 1.6% lower than it was before the reform.

In table 4 we disaggregate the responses of wealth and hours into household types. The largest increase in wealth accumulation is experienced by couples (9.48% relative to the initial steady state).
Table 3: Responses of Aggregate Variables to the Reform

<table>
<thead>
<tr>
<th>Quantity</th>
<th>First Period</th>
<th>Long Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>100.00%</td>
<td>108.7%</td>
</tr>
<tr>
<td>Hours</td>
<td>91.52%</td>
<td>92.04%</td>
</tr>
<tr>
<td>Labor Taxes</td>
<td>201.72%</td>
<td>192.80%</td>
</tr>
<tr>
<td>Output</td>
<td>94.75%</td>
<td>98.40%</td>
</tr>
<tr>
<td>Benefits</td>
<td>94.75%</td>
<td>98.38%</td>
</tr>
<tr>
<td>Net Return</td>
<td>141.65%</td>
<td>132.01%</td>
</tr>
<tr>
<td>Gross Return</td>
<td>92.07%</td>
<td>85.81%</td>
</tr>
</tbody>
</table>

Note: The table expresses aggregate variables in the first period of the transition and the final steady state as a percentage of the variables in the original steady state.

Table 4: Responses of Variables to the Reform

<table>
<thead>
<tr>
<th>Quantity</th>
<th>First Period</th>
<th>Long Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{a}^C$</td>
<td>100.00%</td>
<td>109.48%</td>
</tr>
<tr>
<td>$\bar{a}^{Sm}$</td>
<td>100%</td>
<td>107.10%</td>
</tr>
<tr>
<td>$\bar{a}^{Sf}$</td>
<td>100%</td>
<td>106.12%</td>
</tr>
<tr>
<td>$\bar{a}^{Cm}$</td>
<td>92.47%</td>
<td>92.67%</td>
</tr>
<tr>
<td>$\bar{a}^{Cf}$</td>
<td>91.21%</td>
<td>91.75%</td>
</tr>
<tr>
<td>$\bar{a}^{Sm}$</td>
<td>92.02%</td>
<td>92.64%</td>
</tr>
<tr>
<td>$\bar{a}^{Sf}$</td>
<td>88.86%</td>
<td>90.21%</td>
</tr>
</tbody>
</table>

Note: The table expresses aggregate variables in the first period of the transition and the final steady state, as a percentage of the variables in the original steady state. $\bar{a}$ denotes average assets. $\bar{a}$ is average hours. The superscripts indicate individual and household type. $Cf$ and $Cm$ are married females and males and $Sf$ and $Sm$ are single females and males respectively.

Single households experience a more modest increase in assets. For single male households the increase is in the order of 6.12% and for single females it is 7.1%. Rows 4 to 7 of table 4 report hours for married and single individuals. Married males reduce their work time by roughly 7.3% and married females by 8.25%.  

5.2 The Intrahousehold Allocation

5.2.1 Sharing Rule

The top left panel of figure 2 illustrates the behavior of the male share $\lambda$ over the life cycle in the initial steady state for one household in our simulations. This couple starts with mean productivity in the first period, and at age 40 the female spouse’s productivity drops by about one third, triggering a rise in the male spouse’s share. The share $\lambda$ increases from a half to roughly 0.66. From age 40 until the end of the household’s life, the share remains constant.

To understand this pattern note that though households experience shocks from the first period of their working lives, it is shocks which occur later, that are more likely to make participation constraints bind. Very early shocks are less permanent to the household and arrive at a time when the individuals anticipate to benefit substantially from risk sharing against future changes in labor

\[26\text{Though we leave it out of the table, our findings suggest that in response to the reform there is a change in the non-participation pattern. We find that the fraction of married women that do not work increases to roughly 17% when the reform takes place, as opposed to 10.4% in the original steady state. Therefore the drop in hours for married females represents partly a withdrawal from the labor force. For men, the reduction in hours occurs almost entirely at the intensive margin.}\]
income. Therefore, these shocks are less likely to lead one of the two spouses to renege. On the other hand, by the time the household is at the middle of its working life, and disturbances could be more important, it probably has accumulated enough assets to be able to ward off changes in productivity without any change in $\lambda$. Therefore, in the model households rebargain relatively rarely (twice or three times over the life cycle). Rebargaining may occur in retirement even though income is constant, but only for households where the weight of one of the spouses is too high due to a sequence of very good shocks; as wealth is typically run down in retirement, the less favored spouse seeks to rebargain.

[Figure 2 Here]

In the top right panel of figure 2 we investigate the influence of shocks vs. the life cycle productivity component, on the evolution of the weight. This graph represents the difference of the actual weight, from a path where no shock occurs throughout. There are two noteworthy features: First, as the graph shows, the bulk of the adjustment at age 40 is due to changes in idiosyncratic productivity rather than (changes in) life cycle prices. Second, according to the figure, predictable gender differences in the life cycle paths $L_g(j)$ are factored in the sharing rule at age 25 (there is only a modest increase in the male share of roughly 2 percentage points at age 35). 27

The effects of taxes. The bottom panel of the figure represents with the dashed line the path of $\lambda$ in the final steady state and with the crossed line the analogous path in the first period of the transition. The solid line represents the initial steady state (same as in the top left panel). Notice first that after the reform, the couple at age 25, draws a starting value of the weight that is different from the value in the initial steady state. The reform, therefore, has a differential impact to the male and the female spouses and redistributes bargaining power within the household. We will return to this model feature in a subsequent paragraph. Second, note that in the new steady state with zero capital taxation, the household is better placed to commit to the initial allocation and changes in the share are less. This effect is consistent, with the results of Section 3.1 which suggested that higher wealth (deriving from the reform) improves the household’s risk sharing possibilities. Indeed for the path plotted in the figure the rise in $\lambda$ at age 40 is roughly 8 percentage points in the final steady state, whereas it is roughly 12 percentage points before the reform. 28

Are these changes large enough to impart a big effect on individual consumption and on the intrahousehold allocation? The answer is probably not. To see this note that under non-separable preferences the share of the female spouse on total household consumption is given by \( f \) where

\[
(23) \quad f = \left( \frac{\lambda_2}{1 - \lambda_2} \frac{w_f}{w_m} \right)^{1-\eta(1-\gamma)} - \frac{1}{\gamma}
\]

and \( w_g = wL_g(j)\alpha_g \). According to (23), the female consumption share changes either due to changes in the ratio \( \frac{w_f}{w_m} \), which correspond to the efficient allocation, or due to changes in the sharing rule $\lambda$ which are ex ante inefficient. In the case where the female wage drops by a third in the

27Note that in our calibration which follows Hansen [1993], the productivity path of men rises steeply with age, it peaks at age 45-50 with a ratio of the maximum to initial productivity is 1.22. In contrast, female productivity is relatively flat over the life cycle.

28In considering the first period of the transition, we consider the behavior of $\lambda$ for a couple that is born right when the change in policy takes place. When the couple is of age 40, it has lived for three periods under the new tax regime. Hence this family has had enough time to accumulate wealth in response to the drop in capital taxes.
original steady state, her share of consumption drops by 8.7 percent. Removing the volatility of
the share \( \lambda \) (i.e. imposing \( \lambda = 0.5 \)) reduces the drop to 4.6. Under the tax schedule in the final
steady state the reduction is in the order of 7.9. Therefore in the final steady state there is a benefit
measured in reduced consumption variability but overall it seems quite modest, at least relative to a
full commitment allocation with \( \lambda = 0.5 \). Notice, however, that the effect represented in the figure
is permanent, so it is possible that even a small per period change represents a considerable benefit
for the individual. We will resolve this issue in a later paragraph.

### 5.2.2 Household rebargaining

In order to further investigate, the impact of the reform on intrahousehold commitment, we consider
in table 5 the frequency with which participation constraints bind, before and after the change in
policy. The statistic reported in the table counts the number of times that a change in the weight \( \lambda \)
takes place in a panel of families that is representative of the population. According to the results of
the table, on average 18.34% of households in the old steady state rebargain. Moreover, consistent
with our previous results, most renegotiations occur at young ages (25 -45).

When the change in policy takes place the frequency of rebargaining drops by about 13.5% (3.2
percentage points) in the new steady state. Between ages 25 and 45 marital contracts are renegotiated
nearly 10% less frequently, for households aged 50 to 65 roughly 39% less frequently and for retired
households roughly 78%. The largest percentage drops are therefore experienced by older households.
Our model produces similar implications for the first period of the transition, as is shown in column
3 of the table.

<table>
<thead>
<tr>
<th></th>
<th>Initial</th>
<th>Final</th>
<th>First Period</th>
<th>Only ( \tau_K )</th>
<th>Only ( \tau_N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>23.37%</td>
<td>20.10%</td>
<td>20.55%</td>
<td>16.94%</td>
<td>25.74%</td>
</tr>
<tr>
<td>25-45</td>
<td>38.04%</td>
<td>34.09%</td>
<td>34.61%</td>
<td>29.71%</td>
<td>40.99%</td>
</tr>
<tr>
<td>50-65</td>
<td>6.60%</td>
<td>3.37%</td>
<td>4.25%</td>
<td>1.51%</td>
<td>8.63%</td>
</tr>
<tr>
<td>Retired</td>
<td>1.90%</td>
<td>0.42%</td>
<td>0.44%</td>
<td>0.00%</td>
<td>2.78%</td>
</tr>
</tbody>
</table>

Note: The table shows the frequency with which participation constraints bind, before and after the
reform.

These effects are clearly modest, and it would be surprising if they contributed considerably
towards increasing the intrahousehold risk sharing for the following reasons: First, because, as dis-
cussed previously young households which bargain more frequently, experience only a moderate drop
in rebargaining (in fact there is a rise in rebargaining at age 25 as households start their working lives
with zero wealth). Second, because older households rebargain too little, since they own sufficient
wealth in the initial steady state, and therefore do not benefit from the more substantial loosening
of the limited commitment friction.

**Partial equilibrium effects.** The last two columns of table 5 present the implications of a
partial equilibrium analysis. In the column labeled ‘Only \( \tau_K \)’ we consider setting the capital tax
equal to zero, and keeping all other quantities (taxes, wages, interest rates, etc.) to the original
steady state values. The column labeled ‘Only \( \tau_N \)’ refers to a model where the labor tax increases to
the final steady state value and all other aggregates are kept constant. Two remarks are important:
First, as we noted in Section 3.1, increasing labor taxation may, or may not, lead to less frequently binding participation constraints when $\gamma > 1$. On the one hand, as after-tax income inequality within the household is reduced, rebargaining will probably occur less often; on the other, when total household labor income is lower, the temptation to renge on the contract increases. The results shown in column 5 demonstrate that overall the second channel dominates. Therefore it is only the reduction in $\tau_K$ in the model which improves commitment. Second, these results confirm that accounting for general equilibrium effects is important to not overstate the gains from the reform on the intrahousehold allocation. In section 3.1 we had stated our analysis in partial equilibrium, however preempting that movements in interest rates and wages would mitigate the effects of changes in the tax schedule. The last columns of the table are in line with this intuition.

### 5.2.3 Consumption volatility effects of improved commitment

When shocks to individual labor income occur, individual consumption is affected more if the household sharing rule changes. In the case where $\lambda$ is constant over time, intrahousehold risk sharing is maximized. This case corresponds to a full commitment allocation. When $\lambda$ is updated frequently in response to shocks, intrahousehold insurance is less. We measure the effects of the policy change to the volatility of individual consumption in figure 3. We have computed the conditional standard deviation of the log of consumption for each individual from a panel of 360,000 individuals and averaged by age group. Subsequently we have computed the percentage changes in the volatility in various models relative to the original steady state. The figure reports these changes. 

![Figure 3 Here](image-url)

The solid line represents the change in the final steady state. Notice that individual consumption uncertainty drops considerably over the working life. By age 40 there is a 2% drop in the standard deviation and by age 50 a 4% drop. There are two possible reasons: First, more commitment, under the new policy, improves risk sharing (owing to the fact that families rebargain less frequently over the intrahousehold allocation as we established previously) and second, higher wealth permits to the household and its members to smooth the impact of shocks to consumption (e.g. through precautionary savings).

In order to measure the contribution of improved commitment, in the crossed line we represent the change in consumption volatility when we remove the influence of wealth but allow the household to be able to commit to the initial allocation as in the new steady state. We accomplish this by keeping wealth constant as in the initial steady state and simulating household behavior under the final steady state paths for $\lambda$. The graph indicates that across all age cohorts (with the exception of age 30), more commitment accounts for roughly 50% of the overall drop in consumption variability, the remaining 50% being accounted for by wealth. Notice however, that these changes are not a significant improvement; they do not lead to a large drop in consumption uncertainty for young individuals, who typically face higher consumption variability. For example, the standard deviation of consumption for an age 30 individual is nearly twice as large as at age 60. Therefore, the new

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29Note that the calculation confounds the effect of lower wealth. Higher taxes mean that the typical household is able to accumulate less assets over its working life.

30In retirement there are no shocks and therefore this calculation does not apply.
policy confers only a small improvement in terms of reducing variability when variability is high, and it rather reduces uncertainty when households have already build up wealth so that the overall consumption uncertainty is less.

To emphasize these points in the crossed line in the figure we represent a constant \( \lambda \) (full commitment) allocation with wealth as in the initial steady state. Notice that under full commitment there is a substantial reduction in consumption uncertainty at young ages. We conclude that at least for young households, the policy reform does not bring the allocation closer to the full commitment benchmark.

### 5.2.4 Conditional Volatility of the Sharing Rule

The previous sections showed, through several channels, that lowering capital taxation and shifting the burden towards labor income taxes, may have a modest quantitative effect on the intrahousehold allocation and risk sharing. For example, it was shown analytically in Section 3.1, that households which are more likely to benefit from the reform are wealthy households and that general equilibrium effects are likely to weaken the effects from policy. Moreover, we argued that in a multiperiod life cycle model the economic significance of shocks to the labor income changes depending on the age of the household. Shocks which arrive very early on in the life cycle, are typically less persistent and occur when the household’s horizon is sufficiently long to not discount heavily the future benefits from insurance against further income changes. Therefore, these shocks do not lead to large renegotiations of the marital contract. In contrast, shocks which arrive later on in the life cycle could lead to considerable rebargaining if the household is not sufficiently wealthy.

![Figure 4 Here](image)

In figure 4 we consider how the significance of shocks to the intrahousehold allocation is affected by age and by wealth. We compute for each age group the (next period’s) standard deviation of the sharing rule \( \lambda \) from a representative panel of families, holding their wealth constant. The solid line shows the standard deviation when households are at the borrowing constraint. The dashed line represents the analogous object when initial wealth is set equal to the average of the group which has the lowest fixed effect. The crossed line corresponds to the average wealth in the economy as the initial condition.

The figure confirms that shocks to the labor income lead to more rebargaining when they occur at later stages in the life cycle. It also shows the significance of wealth for the intrahousehold allocation. Young households which are wealth poor (accurately represented by the solid line) can better commit to the allocation than middle aged, equally poor households. But by the middle of the life cycle, when shocks are more important, household wealth is also higher. This is represented by the crossed line. The right panel of the figure shows these graphs in the final steady state. Notice that young households with very little initial wealth, do not gain from the reform in terms of commitment. Wealthier households gain more, but at the same time suffer much less from the limited commitment friction. This analysis explains the modest effects we find in our quantitative model.
5.3 Welfare Evaluation

In table 6, we look at the welfare effects of the change in the tax code. We define our welfare measure as the percentage increment in consumption that keeps expected welfare constant across the two economies (with and without the reform). We report the average value of this quantity for all individuals in our economy, assigning equal weight to each individual in the welfare function, and separately for males and females single and married.\footnote{We construct average utility as follows:}

As the first column shows married households and single female households are on average better off in the final steady state without capital taxation. Single females enjoy the largest welfare gains (2.32\% in terms of consumption). Married men seem to benefit more than married women from the reform (1.04\% vs. 0.49\%). In contrast, single male households are on average worse off in the final steady state. In the first period of the transition (column 2), individuals lose on average from the change in policy, and the largest loses are incurred by single males who are willing to give up 4.62\% of consumption to stay in the old regime.

<table>
<thead>
<tr>
<th>Table 6: Welfare Effects from the Reform</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compensating Final Steady State Transition Variation</td>
</tr>
<tr>
<td>Total</td>
</tr>
<tr>
<td>Married Males</td>
</tr>
<tr>
<td>Married Females</td>
</tr>
<tr>
<td>Single Males</td>
</tr>
<tr>
<td>Single Females</td>
</tr>
</tbody>
</table>

Note: This table shows the percentage increment in consumption that keeps expected welfare constant across the two economies (with and without the reform).

To understand the different welfare responses to the reform across household types, several remarks are important. First, note that single male and single female households, differ mainly in terms

\[
U = \frac{1}{2\mu + (1 - \mu)} \left( \mu \int \sum_g V_g(a, X, \lambda, j) \Gamma_M(da \times dX \times d\lambda \times dj) + \frac{1 - \mu}{2} \int \sum_g S_g(a, X, j) \Gamma_{S,g}(da \times dX \times dj) \right)
\]

where \( \mu \) is the fraction of households populated by couples in our economy (there are 2\( \mu \) married individuals). Since preferences are the same for all individuals the value of the compensation variation is given by:

\[
\left( \frac{U_{\text{tax}}}{U_{\text{notax}}} \right)^{\frac{1}{1 - \gamma}} - 1,
\]

where \( U_{\text{tax}} \) (\( U_{\text{notax}} \)) is average utility in a steady state with (without) capital taxation. Similarly, when we want to make a welfare assessment for married individuals, we compute expected utility as follows:

\[
U_M = \frac{1}{2} \left( \int \sum_g V_g(a, X, \lambda, j) \Gamma_M(da \times dX \times d\lambda \times dj) \right)
\]

where

\[
W = \int M(a, X, \lambda, j) \Gamma_M(da \times dX \times d\lambda \times dj) = \int \lambda V_m(a, X, \lambda, j) + (1 - \lambda)V_f(a, X, \lambda, j) \Gamma_M(da \times dX \times d\lambda \times dj)
\]

Notice that the welfare criterion under 25 is different than the average household utility the way we define it in equation 22. For example, if we were to use the value function \( M(a, X, \lambda, j) \) in our welfare calculation, we would construct average utility for married individuals as:

The value for \( W \) and \( U_M \) do not coincide because the planner attaches a weight equal to 1/2 to every individual, but households attach weights \( \lambda \) and 1 - \( \lambda \) respectively. In the ergodic distribution \( \Gamma_M \), these household weights are generally different from 1/2. Apps and Rees [1988] show that aggregating preferences of individuals into a household utility and maximizing over a policy parameter, can be misleading for policy, because the effects of changes in policies are mediated through the household sharing rule \( \lambda \).
of their respective time paths of life cycle productivity. As discussed previously, in the US data, the productivity path of men rises steeply with age (see the results of Hansen [1993]). Consequently, in the model, men want to borrow against their permanent income to smooth consumption, but cannot do so due to the presence of the borrowing constraints. When capital taxes are replaced by labor taxes, young single men experience a fall in income which makes their consumption path even steeper over the life cycle.\textsuperscript{32} Female productivity, on the other hand, is not steeply rising; therefore, young single women in the model are not made as worse off when the policy changes.

Second, consider the case of a married household. As most of the married household’s income comes from the husband, couples are more like single men than single women. That is to say, the couple cannot to a great effect smooth the life cycle income profile by allocating more hours to the wife at the early stages of the working life, and therefore in terms of the welfare outcomes the gender differences in life cycle labor income are possibly not important.

However, couples are different from singles in one other respect. They are at least partially able to insure against fluctuations in labor income through joint adjustments in labor supplies and through transfers from one spouse to the other. These transfers are substantial in the model even in the presence of the limited commitment friction, and weaken the incentive of couples to self-insure against income shocks via precautionary savings.\textsuperscript{33} In this case, we can make the argument that high capital taxation should be more distortive to couples than to single households since as is well known, a strong demand for precautionary savings, is associated with a smaller distortionary impact of capital taxation (see Domeij and Heathcote [2004]).\textsuperscript{34} We illustrated in a previous section that after the reform, there is a stronger increase in capital accumulation in dual earner households. The wealth of married households increases by 9.5\% in the final steady state whereas the average increase in single households was in the order of 6.6\%. Therefore, through this channel couples benefit more from the reform.

Such an effect ought to be larger, the closer the intrahousehold allocation is to full commitment (in this case transfers are maximized). We have confirmed with numerical simulations from a full commitment model that couples indeed experience an even stronger increase in wealth accumulation, in response to the reform, and slightly larger welfare gains. It should therefore be understood that the larger increase in wealth in couples is related to the differences in the incentives to hold precautionary savings between singles and couples, and has little to do with the effects of taxes on the limited commitment friction.\textsuperscript{35} Moreover, if the new policy had a large impact on the

\textsuperscript{32}See Erosa and Gervais [2002] for an analysis of the effect of life cycle income profiles on the preferences over labor and capital taxes.

\textsuperscript{33}We calculate in the initial steady state income subsidies from one household member to the other as a fraction of total household resources devoted to finance consumption. We define the transfer as the excess of private consumption over individual income, assuming that each spouse owns half of the household wealth stock each period and finances half of the wealth brought forward to the next period. We find that intrahousehold transfers are roughly 13\% of total consumption spending. Moreover, transfers are largest for young households and smaller for retired households due to two reasons: First because as discussed previously in the model the initial allocation favors the wife, and is rebargained as the household ages, and second because shocks to the labor income which are offset by transfers, do not occur during retirement.

\textsuperscript{34}The argument is that with high capital taxation more of the household’s resources are made out of risky labor income, which stimulates the demand for precautionary savings. When capital taxes are eliminated households accumulate wealth due to the higher return, but lose the strong incentive for precautionary savings, since capital income, at least in the model, is riskless.

\textsuperscript{35}In the Appendix we make this point utilizing the two period version of our model. We show that limited commitment has an ambiguous effect on household savings.
intrahousehold allocation in our model, in the sense of enhancing commitment within the household, we would anticipate to find considerably larger welfare gains in couples than in single households. However, our analysis shows that there is only a modest impact which does not add much to the welfare calculation. Instead couples experience gains and loses from the reform which are well within those experienced by single households. Our results in this section reinforce our conclusion that the effect of changes in capital and labor taxes on the intrahousehold allocation and risk sharing are not substantial.

5.3.1 Division of welfare gains and loses within the household

Married women do better than married men according to the welfare measure in table 6. This feature of the model may seem surprising in light of the fact single women do better than single men after the reform. It is worth emphasizing, however, that the outside option of married women exhibits a different welfare effect to the reform, than the analogous effect for single women, owning to the fact that the wealth profiles are different between singles and couples. Put differently, in order to evaluate how the outside options of married men and women respond to the reform it does not suffice to look at the welfare responses of single households.

To explain why married women do worse than married men, we note that after the reform, couples are better able to commit to the initial allocation. This is a crucial observation because in our calibration we gave to women a large initial share (low \( \lambda \)) in order to match average hours as in the data. Consequently, in the model married men seek to re-bargain after a few periods when male relative to female life cycle productivity grows. The expectation of this future re-bargaining makes men, in the initial steady state, willing to tolerate a lower starting value of \( \lambda \). When the tax code changes, however, and commitment to the initial allocation improves, married men anticipate a smaller renegotiation of the contract in the future. In effect, they want an increase in \( \lambda \) at the matching stage to compensate. Consistent with this view our model generates an increase in the average male share at the initial allocation of roughly one percentage point.

5.3.2 Welfare gains from commitment

Our theory predicts that lower capital taxation, affects intrahousehold risk sharing and lowers individual consumption uncertainty. In this paragraph we investigate whether the improvement in insurance possibilities, we have documented previously, generates welfare gains to households. In order to isolate the effect of more commitment from changes in the tax code and the wealth paths, we solve for the gains in commitment keeping household wealth constant as in the old steady state. We consider the following cases: 1. the household is able to commit as in the final steady state (that is the paths of \( \lambda \) are those of the final steady state), 2. the household can commit as in the old steady state (the paths of \( \lambda \) are those of the old steady state) and 3. the household can fully commit (\( \lambda \) is constant over time). To calculate the welfare benefit from improved commitment we compute the average value of the compensating variation between 2. and 1. In order to quantify how far this welfare gain is from the full commitment allocation we compare 2. and 3.

Our results are as follows: First we find that the welfare gain from improved commitment in the final steady state is tiny, roughly 0.062% of consumption. Second, we obtain a welfare gain from full commitment in our model that is substantial, roughly 0.82% of consumption. These results
imply that the shifts in the tax schedule we consider have only a small effect on the intrahousehold allocation.

5.4 Robustness and Discussion

5.4.1 Log Separable Utility

We briefly discuss the effects of the reform when utility is separable in consumption and leisure. In this version of our model we keep all of the parameters as in the benchmark. \(^{36}\) We find that in the old steady state, families rebargain 10.47% on average. This fraction is 12.58% for ages 25-45, 11.30% between ages 50 and 65 and 0.26% in retirement. \(^{37}\) In the new steady state, overall rebargaining drops to 9.83%. For the age groups we obtain 13.06%, 8.48% and 0.05% respectively. These results are in line with our previous analysis of non-separable utility, which suggests that the exact specification of preferences is not important for our quantitative results. We conclude that the effect of taxation on the intrahousehold allocation is limited.

5.4.2 The (Potential) Effects of the Tax Code on the Decision to Divorce

Our theoretical results were derived from a model in which couples do not divorce in equilibrium. This assumption which is rather common in the literature (see for example Alesina et al. [2011], Guner et al. [2012a] and Guner et al. [2012b]), is basically made to isolate our focus on the effects of the tax code on the limited commitment friction within the marriage. However, we wish in this section to briefly describe potential effects of the tax schedule on the decision to divorce, if we were to give this option to the family.

We begin with two period model of section 3. In that model a couple would not divorce as long as \(\xi > 0\), since, as discussed previously, it is always possible in the second period to give to the spouses what they would get if they were bachelors (i.e. \(\frac{A}{2} + \epsilon g w(1 - \tau_N)\)). Divorces may occur in that model if \(\xi\) in period 2 can be negative. \(^{38}\) We describe here how changes in the value of \(\xi\) over time, which motivate endogenous divorces, affect our results.

To simplify assume that preferences are log-log. If the couple draws a random value \(\xi_2\) in period 2 we can derive the updating rule for the share \(\lambda\) as:

\[
\lambda_2 \in \left\{ e^{-\xi_2} \frac{A}{A_e + \sum g \epsilon gw(1 - \tau_N)} \cdot 1 - e^{-\xi_2} \frac{A}{A_e + \sum g \epsilon gw(1 - \tau_N)} \right\}
\]

for \(\xi_2 \geq 0\). Note that if the couple were to divorce (with \(\xi_2 < 0\)) the above equation would not hold; then the tax schedule has effectively no effect on the intrahousehold allocation as divorce would break the commitment. Moreover, since the decision to divorce solely hinges on the value of \(\xi_2\) changes in taxes have no impact on this margin. The tax schedule cannot make the marital surplus positive and sustain the marriage.

\(^{36}\)Since the specification of individual utility is different, the model requires a different value of \(\xi\). As previously we choose this value to give us as much rebargaining as possible.

\(^{37}\)Note that with higher labor taxes in the life cycle model rebargaining can increase for young households because taxes reduce labor income and wealth accumulation in the first few periods of the working life. This implication therefore, does not contradict our analytical result that higher taxes under log separable utility improve commitment.

\(^{38}\)This is basically the way divorces are modelled in Voena [2012].
This however, does not mean that shocks which produce endogenous divorces do not have a potentially important interaction with tax rates. To see this consider the case of a low value of $\xi_2$. Then the limited commitment becomes more severe (the sharing rule being more volatile) when $\epsilon_m > \epsilon_f$ or $\epsilon_f > \epsilon_m$ (i.e. when the participation constraints may bind). In this case having higher wealth would be useful to insulate the share $\lambda_2$ from the shocks to the labor income and to the value $\xi_2$.

The following observation is important: Notice that from (26) it is easy to show that wealth and taxes have a bigger effect the closer $\xi_2$ is to zero. Conversely, if $\xi_2$ is large, the limited commitment problem is not severe, and changes in policy will not affect risk sharing substantially. In the quantitative model we have set $\xi$ to the lowest possible value to not have divorces in equilibrium. By this reasoning our results exacerbate the effect of the limited commitment friction and the effects of policy. 39

Finally it is worth noting that under the multiperiod model we should not anticipate that taxes are neutral with respect to the decision to divorce. To investigate how the change in the tax schedule impacts the divorce decision in the quantitative model we have conducted the following experiment: We solved the model with a lower value of $\xi$ (relative to our benchmark calibration) and looked at the marital surplus from the value functions. The numerical solution to the model suggested that when the policy changes (and capital taxes are eliminated) the regions in the state space over which divorce is optimal, widens 40. Therefore, it seems that if we were to endogenize divorce in the model, we would obtain a rise in divorce rates in equilibrium. Though we acknowledge that such effects are not trivial, we note that a higher divorce rate goes against the notion that lowering capital taxation generates insurance gains for married households. It therefore does not contradict our conclusions.

6 Conclusion

In this paper, we study the welfare effects of a reform that eliminates capital taxation, in a model with gender and marital status heterogeneity, uncertain labor income and incomplete financial markets. Decision making within the couple is represented as a contract under limited commitment. When the labor income of one spouse increases, the household must allocate more weight to her well-being. The household gives up some risk sharing to satisfy a participation constraint. We investigate how the tax code affects the intrahousehold allocation and especially the risk sharing possibilities of the couple. We show that lower capital taxes, lead to wealth accumulation and more insurance within the family. Our theory motivates this implication through the empirical observation that wealth is a commonly held resource within US households. On the other hand we show that higher labor taxes, have an ambiguous effect; they make the distribution of disposable income less dispersed and therefore alleviate the limited commitment problem, but they also reduce the overall household resources and may therefore make it more tempting to renege on past commitments.

39To put this differently if we were to assume $\xi$ is random and with a stochastic process which would enable us to match the divorce rates we see in the data, a smaller fraction of (marginal) families would be infected with the limited commitment friction than in the quantitative model with the constant and low $\xi$. Another way of saying this is that it is not the shock to $\xi$ which is important, rather the interaction between $\xi$ and the productivity shocks is important as equation (26) reveals.

40This is so because the rise in labor taxes has an adverse effect on commitment for younger households. It is consistent our previous results.
In our quantitative life cycle model, we find that when the policy changes, the new intrahousehold allocation features less rebargaining, and therefore more insurance, but the overall effect is small. The reason for this is that young households, which typically do not own sufficient assets, do not benefit from the improved commitment, and that old households, which theoretically could stand to gain considerably from the policy change, have sufficient wealth even under the old tax schedule, to effectively ward off the effects of the limited commitment friction.

Our analysis in this paper extends to a dynamic model, the literature on optimal taxation within the so called collective framework of intrahousehold decisions. We utilize our model to illustrate how the optimal behavior of the household changes over the life cycle, and how it is shaped by the tax code. Though we document a small impact, considering the effects of a shift in essentially a linear tax schedule, we feel that there is scope to further investigate the impact of taxes on the intrahousehold allocation. For example, in the case of nonlinear taxation of labor income it is obvious that the effects from policy on the outside options of individuals and on the distribution of resources within the family, could be substantial. These issues have been thoroughly investigated in static models. We are convinced that the model proposed in this paper can be used to study the dynamic effects of such policies.

References


A Appendix

A.1 Derivations in the Two Period Model

In this section, we derive explicitly the formulas from the two period model of section 3 that were omitted from the text. We assume that household members value the consumption-leisure bundle according to a utility function of the form \( \frac{(c_2^{1-(1-\gamma)})^{1-\gamma}}{1-\gamma} \). The optimal consumption and leisure choices in period 2 satisfy:

\[
\lambda_2 \eta c_m^{(1-\gamma)-1} l_m^{(1-\eta)(1-\gamma)} = (1 - \lambda_2) \eta c_f^{(1-\gamma)-1} l_f^{(1-\eta)(1-\gamma)} \quad \text{and} \quad \frac{(1-\eta)c_g}{\eta l_g} = w(1 - \tau_N)\epsilon_g
\]

Assuming an interior solution, we can substitute the intraperiod consumption-leisure optimality condition into the consumption first-order condition and obtain:

\[
\lambda_2 \frac{c_m^{-\gamma}}{(w(1 - \tau_N)\epsilon_m)^{(1-\eta)(1-\gamma)}} = (1 - \lambda_2) \frac{c_f^{-\gamma}}{(w(1 - \tau_N)\epsilon_f)^{(1-\eta)(1-\gamma)}}
\]

which gives that female consumption is \( f(\lambda_2, \epsilon) = (\frac{\lambda_2}{1 - \lambda_2}) \frac{(\epsilon_f)^{(1-\eta)(1-\gamma)}}{(\epsilon_m)^{(1-\eta)(1-\gamma)}} \) times male consumption in the model. Solving for the optimal choices, we can write male and female utility as:

\[
\left( \frac{A_c + \sum_g w(1 - \tau_N)\epsilon_g}{(1 + f(\lambda_2, \epsilon)) (w(1 - \tau_N)\epsilon_m)^{(1-\eta)}} \right)^{1-\gamma} \frac{\chi}{1-\gamma} + \xi \geq \left( \frac{\frac{A_c}{f} + \frac{w(1 - \tau_N)\epsilon_m}{(w(1 - \tau_N)\epsilon_m)^{(1-\eta)}}}{A_c + \sum_g w(1 - \tau_N)\epsilon_g} \right)^{1-\gamma} \frac{\chi}{1-\gamma}
\]

\[
\left( \frac{f(\lambda_2, \epsilon)}{(1 + f(\lambda_2, \epsilon))} \right)^{-1-\gamma} \leq \frac{1}{(1 + f(\lambda_2, \epsilon))} \leq \xi \left( \frac{(w(1 - \tau_N)\epsilon_m)^{(1-\eta)}}{A_c + \sum_g w(1 - \tau_N)\epsilon_g} \right)^{1-\gamma} + \left( \frac{\frac{A_c}{f} + \frac{w(1 - \tau_N)\epsilon_f}{(w(1 - \tau_N)\epsilon_f)^{(1-\eta)}}}{A_c + \sum_g w(1 - \tau_N)\epsilon_g} \right)^{1-\gamma}
\]

where \( \chi = (\frac{\eta^n(1-\eta)^{1-\gamma}}{1-\gamma}) \). Rearranging, we can express the sharing rule with the following nonlinear equations:

\[ 27 \quad \left( \frac{1}{(1 + f(\lambda_2, \epsilon))} \right)^{1-\gamma} \leq \frac{\frac{A_c}{f} + \frac{w(1 - \tau_N)\epsilon_m}{(w(1 - \tau_N)\epsilon_m)^{(1-\eta)}}}{A_c + \sum_g w(1 - \tau_N)\epsilon_g} \]

\[ 28 \quad \left( \frac{f(\lambda_2, \epsilon)}{(1 + f(\lambda_2, \epsilon))} \right)^{-1-\gamma} \leq \frac{\frac{A_c}{f} + \frac{w(1 - \tau_N)\epsilon_f}{(w(1 - \tau_N)\epsilon_f)^{(1-\eta)}}}{A_c + \sum_g w(1 - \tau_N)\epsilon_g} \]

The effect of changes in labor taxes. Equations 27 and 28 define the upper and the lower bound that the weight \( \lambda_2 \) needs to respect in order for the participation constraints to be satisfied. The partial derivatives of the right hand side of equation 27 with respect to \((1 - \tau_N)\) are:

\[ 29 \quad (1 - \gamma) \left[ \left( \frac{\frac{A_c}{f} (1-\eta)}{1 - \tau_N} - \eta \sum_g w \epsilon_g \right) \kappa_1(A_c, \epsilon) + (A_c w (\epsilon_m - \frac{\sum_g \epsilon_g}{2})) \kappa_2(A_c, \epsilon) \right] \]

where \( \kappa_1 = \frac{(w(1 - \tau_N)\epsilon_g)^{(1-\eta)(1-\gamma)}}{(A_c + \sum_g w(1 - \tau_N)\epsilon_m)^{1-\gamma}} > 0, \kappa_2 = \frac{\frac{A_c}{f} + \frac{w(1 - \tau_N)\epsilon_m}{(w(1 - \tau_N)\epsilon_m)^{(1-\eta)}}}{(A_c + \sum_g w(1 - \tau_N)\epsilon_g)^{1-\gamma}} > 0. \) The derivative of equation 28 is similar and for the sake of brevity omitted. As discussed in text, 29 illustrates that the effect of changes in labor taxation on the household sharing rule depends on the level of wealth of the household \( A_c \). If \( A_c = 0 \), it reduces to:

\[ 30 \quad - (1 - \gamma) \left[ \xi \eta \sum_g w \epsilon_g \right] \kappa_1(A_c, \epsilon) > 0 \]

As the LHS of 27 is decreasing in \( \lambda_2 \), a positive partial derivative means that \( \lambda_2 \) has to increase by less
to satisfy the participation constraint of the male spouse. Therefore, in the case of $\epsilon_m >> \epsilon_f$ where the lower bound applies, the disturbance to the household sharing rule is smaller. The converse may obtain if $A_c > 0$. As in the case of log separable utility, an overall negative derivative 29 yields that lowering labor taxes makes it more difficult for household members to commit to an allocation.

The effect of changes in capital taxes. We derive the partial derivative of equation 27 with respect to financial income as:

$$(\gamma - 1) \left[ \xi \left( \frac{(w(1 - \tau_N)\epsilon_m)^{(1 - \eta)}}{A_c + \sum g w(1 - \tau_N)\epsilon_g} \right)^{1 - \gamma} + \left( \frac{\frac{A_c}{2} + w(1 - \tau_N)\epsilon_m - \gamma}{(A_c + \sum g w(1 - \tau_N)\epsilon_g)^{2 - \gamma}} \right) (\epsilon_m - \frac{\sum g \epsilon_g}{2}) \right]$$

The leading term in 31 is always positive, meaning that an increase of financial income relaxes the constraint. The second term is positive only when $\epsilon_m > \epsilon_f$. Since equation 27 defines the lower bound on $\lambda_2$, it binds only when male productivity exceeds female productivity. Therefore, higher financial wealth or lower capital taxation enhance the households commitment. This proves proposition 3 in the main text.

Nash Bargaining. We derive the impact of wealth on the intrahousehold allocation under the assumption that couples bargain each period using a Nash protocol. We argued in text that under Nash bargaining, the period 2 allocation $\lambda_2$ solves the following product:

$$(\lambda_2^*(a, \epsilon) \in \arg\max_{\lambda_2} \left[ \left( \frac{A_c + \sum g w(1 - \tau_N)\epsilon_g}{(1 + f(\lambda_2, \epsilon))(w(1 - \tau_N)\epsilon_m)^{1 - \eta}} \right)^{1 - \gamma} \frac{\chi}{1 - \gamma} + \xi - \left( \frac{\frac{A_c}{2} + w(1 - \tau_N)\epsilon_m}{(w(1 - \tau_N)\epsilon_m)^{1 - \eta}} \right)^{1 - \gamma} \frac{\chi}{1 - \gamma} \right]$$

The first order derivative with respect to $\lambda_2$ in 32 leads to the following optimality condition:

$$\frac{\Phi_m^{1 - \gamma}}{\Omega_m} = \frac{\Phi_f^{1 - \gamma}}{f(\lambda_2^*, \epsilon)\Omega_f}$$

where:

$$\Phi_m = \left( \frac{A_c + \sum g w(1 - \tau_N)\epsilon_g}{(1 + f(\lambda_2^*, \epsilon))(w(1 - \tau_N)\epsilon_m)^{1 - \eta}} \right)^{1 - \gamma} \frac{\chi}{1 - \gamma} + \xi - \left( \frac{\frac{A_c}{2} + w(1 - \tau_N)\epsilon_m}{(w(1 - \tau_N)\epsilon_m)^{1 - \eta}} \right)^{1 - \gamma} \frac{\chi}{1 - \gamma}$$

$$\Omega_m = \left( \frac{A_c + \sum g w(1 - \tau_N)\epsilon_g}{(1 + f(\lambda_2^*, \epsilon))(w(1 - \tau_N)\epsilon_m)^{1 - \eta}} \right)$$

$$\Phi_f = \left( \frac{f(\lambda_2^*, \epsilon)(A_c + \sum g w(1 - \tau_N)\epsilon_g)}{(1 + f(\lambda_2^*, \epsilon))(w(1 - \tau_N)\epsilon_f)^{1 - \eta}} \right)^{1 - \gamma} \frac{\chi}{1 - \gamma} + \xi - \left( \frac{\frac{A_c}{2} + w(1 - \tau_N)\epsilon_f}{(w(1 - \tau_N)\epsilon_f)^{1 - \eta}} \right)^{1 - \gamma} \frac{\chi}{1 - \gamma}$$

$$\Omega_f = \left( \frac{f(\lambda_2^*, \epsilon)}{(1 + f(\lambda_2^*, \epsilon))(w(1 - \tau_N)\epsilon_f)^{1 - \eta}} \right)$$

In order to obtain the derivative $\frac{d\lambda_2}{da_c}$ we utilize the Implicit Function Theorem. The derivative
Note that in the case where \( M(37) \) is satisfied:

\[
- \chi \left( \frac{\Phi_{m}^{1-\gamma}}{\Omega_{m}} (A_{c} + \sum_{g} w(1 - \tau_{N})\epsilon_{g}) (1 - \frac{\left( \frac{A_{c}}{2} + \epsilon_{m} w(1 - \tau_{N}) \right)^{1-\gamma}}{\Phi_{m}^{1-\gamma} \left( \epsilon_{m} w(1 - \tau_{N})^{(1-n)(1-\gamma)} \right)} + \frac{\chi}{\Omega_{f}} (A_{c} + \sum_{g} w(1 - \tau_{N})\epsilon_{g}) (1 - \frac{\left( \frac{A_{c}}{2} + \epsilon_{f} w(1 - \tau_{N}) \right)^{1-\gamma}}{\Phi_{f}^{1-\gamma} \left( \epsilon_{f} w(1 - \tau_{N})^{(1-n)(1-\gamma)} \right)} \right)
\]

\[= -\gamma \frac{f_{\lambda_{2}}^{*}}{f(\lambda_{2}^{*}, \epsilon)} \left( \frac{\Phi_{m}^{1-\gamma}}{f(\lambda_{2}^{*}, \epsilon) \Omega_{f}} \left( \frac{f_{\lambda_{2}}^{*}}{f(\lambda_{2}^{*}, \epsilon) (1 + f(\lambda_{2}^{*}, \epsilon))} - \frac{\Phi_{m}^{1-\gamma}}{f(\lambda_{2}^{*}, \epsilon) \Omega_{m}} \left( 1 + f(\lambda_{2}^{*}, \epsilon) \right) \right) \right) \]

where \( f_{\lambda_{2}} \) represents the partial derivative. Note that for the sake of brevity the detailed derivations of A.1 are omitted. Moreover, note that since the sign of the bottom term in A.1 is positive by the properties of \( f \) of A.1 are omitted. Moreover, note that since the sign of the bottom term in A.1 is positive by the properties of \( f_{\lambda_{2}} \) the sign of \( \frac{d\lambda_{2}}{d\lambda_{c}} \) is basically the sign of the top lines in A.1. These terms may be written as follows:

\[
- \chi \left( \frac{\Phi_{m}^{1-\gamma}}{\Omega_{m}} (A_{c} + \sum_{g} w(1 - \tau_{N})\epsilon_{g}) (1 - \frac{\left( \frac{A_{c}}{2} + \epsilon_{m} w(1 - \tau_{N}) \right)^{1-\gamma}}{\Phi_{m}^{1-\gamma} \left( \epsilon_{m} w(1 - \tau_{N})^{(1-n)(1-\gamma)} \right)} + \frac{\chi}{\Omega_{f}} (A_{c} + \sum_{g} w(1 - \tau_{N})\epsilon_{g}) (1 - \frac{\left( \frac{A_{c}}{2} + \epsilon_{f} w(1 - \tau_{N}) \right)^{1-\gamma}}{\Phi_{f}^{1-\gamma} \left( \epsilon_{f} w(1 - \tau_{N})^{(1-n)(1-\gamma)} \right)} \right)
\]

\[= -\gamma \left( \frac{\Phi_{m}^{1-\gamma}}{f(\lambda_{2}^{*}, \epsilon) \Omega_{f}} \left( \frac{f_{\lambda_{2}}^{*}}{f(\lambda_{2}^{*}, \epsilon) (1 + f(\lambda_{2}^{*}, \epsilon))} - \frac{\Phi_{m}^{1-\gamma}}{f(\lambda_{2}^{*}, \epsilon) \Omega_{m}} \left( 1 + f(\lambda_{2}^{*}, \epsilon) \right) \right) \right) \]

where \( U_{j,g}, j \in \{ s, m \} \) is the utility of the spouse of gender \( g \) as a single \( (j = s) \) or under the marriage (subscript \( j = m \)). Making use of the optimality condition of program 32 we can write:

\[
\frac{\chi}{\Omega_{m}} (A_{c} + \sum_{g} w(1 - \tau_{N})\epsilon_{g}) (1 - \frac{\left( \frac{A_{c}}{2} + \epsilon_{m} w(1 - \tau_{N}) \right)^{1-\gamma}}{\Phi_{m}^{1-\gamma} \left( \epsilon_{m} w(1 - \tau_{N})^{(1-n)(1-\gamma)} \right)} + \frac{\chi}{\Omega_{f}} (A_{c} + \sum_{g} w(1 - \tau_{N})\epsilon_{g}) (1 - \frac{\left( \frac{A_{c}}{2} + \epsilon_{f} w(1 - \tau_{N}) \right)^{1-\gamma}}{\Phi_{f}^{1-\gamma} \left( \epsilon_{f} w(1 - \tau_{N})^{(1-n)(1-\gamma)} \right)} \right)
\]

\[= -\gamma \left( \frac{\Phi_{m}^{1-\gamma}}{f(\lambda_{2}^{*}, \epsilon) \Omega_{f}} \left( \frac{f_{\lambda_{2}}^{*}}{f(\lambda_{2}^{*}, \epsilon) (1 + f(\lambda_{2}^{*}, \epsilon))} - \frac{\Phi_{m}^{1-\gamma}}{f(\lambda_{2}^{*}, \epsilon) \Omega_{m}} \left( 1 + f(\lambda_{2}^{*}, \epsilon) \right) \right) \right) \]

Note that in the case where \( \epsilon_{m} = \epsilon_{f} \), \( \lambda_{2}^{*} = \frac{1}{2} \) and \( f(\lambda_{2}^{*}, \epsilon) = 1 \). In this case 36 equals zero because the utility that either spouse gets from the marriage (net of \( \xi \)) is precisely the utility they get as singles). If however, \( \epsilon_{m} > \epsilon_{f} \) then \( \lambda_{2}^{*} > \frac{1}{2} \) and \( f(\lambda_{2}^{*}, \epsilon) < 1 \). In this case the male spouse gets a higher utility from being married because his bargaining power (relative to the outside option) increases. This makes the leading term in brackets negative. Analogously with \( f(\lambda_{2}^{*}, \epsilon) < 1 \) the second (positive) term in the brackets is reduced, though positive. Overall, the derivative \( \frac{d\lambda_{2}}{d\lambda_{c}} \) is negative. To put it differently when \( \lambda_{2}^{*} > \frac{1}{2} \), higher wealth reduces the value and when \( \lambda_{2}^{*} < \frac{1}{2} \) the opposite holds; there is a rise in \( \lambda_{2}^{*} \) with higher wealth.

**A.2 Home Production**

We consider the implications of home production, which requires household members’ time. In the two-period model we illustrate first what this implies for the specialization of labor within the household, the intrahousehold allocation and commitment. Then we analyze how these depend on the tax code.

As a simplifying assumption, following Knowles (2007)), there is a subsistence requirement in home production \( \bar{x} \) and agents do not derive any further utility from home production. The amount required for a single household, \( \bar{x}_{s} \), may be different to what is required in a couple household, \( \bar{x}_{M} \). The home good is produced using male and female time, \( h_{m} \) and \( h_{f} \) respectively, which are perfect substitutes, according to \( x = h_{m} + h_{f} \). Leisure time is therefore \( l_{g} = 1 - n_{g} - h_{g} \) for \( g \in \{m, f\} \).

To characterize the households program we solve backwards. Given the wealth endowment and the levels of productivity, \( M_{2}(a_{1}, \lambda_{2}, \epsilon) \) is a solution to:

\[
M_{2}(a_{1}, \lambda_{2}, \epsilon) = \max_{c_{2}^{m}, c_{2}^{f}} \lambda_{2} u(c_{2}^{m}, l_{2}^{m}) + (1 - \lambda_{2}) u(c_{2}^{f}, l_{2}^{f}) + \xi
\]
subject to:

\[
\sum_g c_2^g = \sum_g n_2^g w(1 - \tau_N)\epsilon_g + a_1(1 + r(1 - \tau_K))
\]

\[
\bar{x}_M = h_m + h_f
\]

\[
l_g = 1 - n_g - h_g
\]

\[
0 < l_g < 1, 0 < n_g < 1, 0 < h_g < 1 \text{ for } g \in \{m, f\}
\]

Provided that \(\bar{x}_M\) is sufficiently small, the optimal allocation is such that only the spouse with the lower wage rate, i.e. the secondary earner, produces the home good. The optimal allocation satisfies \(\lambda_2 u_c(c_2^m, l_m^m) = (1 - \lambda_2) u_c(c_2^f, l_f^f)\) and \(\frac{w(c_2^m, l_m^m)}{w(c_2^f, l_f^f)} = w(1 - \tau_N)\epsilon_g\). The relative allocation of consumption and leisure is the same as in the baseline model, since the secondary earner who shoulders the home production is compensated by having to work fewer hours in the market, leaving relative leisure unaffected. However, there is a level effect as with home production the equilibrium budget constraint is

\[
\sum_g c_2^g = \sum_g (1 - l_g^2)w(1 - \tau_N)\epsilon_g + a_1(1 + r(1 - \tau_K)) - \bar{x}_M w(1 - \tau_N)\epsilon_f
\]

where \(f\) denotes the secondary earner, i.e. \(f = m\) if \(\epsilon_m < \epsilon_f\) and \(f = f\) otherwise.

Comparing equations (42) and (1) in the main text shows that the need for home production is isomorphic to lowering the time endowment of the secondary earner by \(\bar{x}_M\), which in turn is isomorphic to lowering the households wealth by \(\bar{x}_M w(1 - \tau_N)\epsilon_f\).

Apart from this negative wealth effect nothing changes compared to the baseline model, since couples can decide period-by-period who shoulders the home production. The participation constraints that need to be satisfied are:

\[
u(c_2^g, l_2^g) + \xi \geq S_g(D_g(a_1), \epsilon_g) \quad g \in \{m, f\}
\]

where, as before, \(S_g(D_g(a_1), \epsilon_g)\) is the utility of the household member of gender \(g\) if the marriage breaks up, in which case they receive a fraction \(D_g(a_1)\) of family wealth, but notice that the single household too has to devote some time to home production, \(h_g = \bar{x}_S\). Therefore

\[
S^2(a_1, \epsilon) = \max_{c_2, l_2} u(c_2, l_2)
\]

subject to:

\[
c_2 = (1 - \bar{x}_S - l_2)w(1 - \tau_N)\epsilon + a_1(1 + r(1 - \tau_K))
\]

The effects of capital taxes are therefore the same as in the baseline model. However, the implications of labor taxes differ slightly. First, notice that a change in labor taxation will have no effect on the pattern of specialization within the household as it only depends on relative wages. Second, note that a reduction in labor taxes increases the time loss of the secondary earner due to home production and acts as if the household’s wealth was reduced. Through this channel it reduces intra-household insurance, abstracting from the effect through the home production requirement on \(S_g\) for the moment. (However, lowering \(\tau_N\) also acts as a negative wealth effect on single households and if \(\bar{x}_S\) is big the bargaining set may increase, which would tend to increase risk-sharing possibilities.)

### A.3 Optimal Savings

In this section, we investigate what determines a couple household’s savings. Since our focus is on the intertemporal behavior of families we want to assess whether limited commitment affects the households demand for assets. Moreover, given our results that policies that lower capital taxation lead to gains in terms of commitment we want to investigate here whether more commitment is an
additional channel that encourages asset accumulation in the model. To preview our results we find that household bargaining has an impact on the savings schedule but that it is impossible to sign its effect, meaning that over some parts of the state space more commitment may encourage savings but may discourage them in other parts.

Consider a couple that starts the first period of the life cycle with a weight \( \lambda_1 = \frac{1}{2} \). Note that an increase in savings in the first period entails a cost in terms of marginal utility. If utility is log-log separable, this cost is given by the expression \( \frac{1}{\lambda_1(1-\tau_N)} \), and if \( \gamma > 1 \) it is given by \( \frac{-a_1 + 2w(1-\tau_N)}{(w(1-\tau_N))(1-\eta)(1-\gamma)} \chi \) where \( \chi = \eta(1-\gamma)(1-\eta)(1-\gamma) \). The second period benefit from higher savings is given by:

\[
\beta(1 + r(1-\tau_K)) E_1 \left[ \lambda_2 \frac{du_m}{dA_c} + (1 - \lambda_2) \frac{du_f}{dA_c} + \frac{d\lambda_2}{dA_c}(u_m - u_f) \right]
\]

The leading two bracketed terms in (46) represent the marginal benefit, keeping the household sharing rule constant, whereas the last term measures the effect of higher savings on the sharing rule. If the couple was able to commit to the first period contract, setting \( \lambda_2 = \frac{1}{2} \) everywhere on the state space, the derivative \( \frac{d\lambda_2}{dA_c} \) would equal zero; the marginal utility terms would be the only ones that would count for the marginal benefit of household savings.

Under log separable utility, we can show that

\[
E_1 \left[ \lambda_2 \frac{du_m}{dA_c} + (1 - \lambda_2) \frac{du_f}{dA_c} \right] = \frac{1}{\sum g w(1-\tau_N) + A_c}
\]

Notice that the right hand side of (47) is independent of the weight \( \lambda_2 \). In fact, this expression is the same as the one we would get if we were to solve for the full commitment allocation that sets \( \lambda_2 = \frac{1}{2} \) regardless of the productivity levels of the male and female spouse. Under log utility, therefore, it is the last term in (46) \( (\frac{d\lambda_2}{dA_c}(u_m - u_f)) \) that describes the impact of limited commitment on the households savings. We therefore have to focus on that term.

We consider separately each relevant region in the state space where \( \frac{d\lambda_2}{dA_c}(u_m - u_f) \) is different from zero, that is every region where the marital contract is rebargained. As discussed previously, for male productivity \( \epsilon_m \) less than a lower bound \( \epsilon_m(A_c, \epsilon_f) \) the weight \( \lambda_2 \) will fall to \( \lambda_2^U \) (there is an increase in the female spouse’s share). Conversely, if \( \epsilon_m > \epsilon_m(A_c, \epsilon_f) \) (upper bound) then \( \lambda_2 = \lambda_2^L \). In any other region there is no rebargaining of the households allocation and, therefore, \( \lambda_2 = \frac{1}{2} \).

Thus we can write the conditional expectation of \( \frac{d\lambda_2}{dA_c}(u_m - u_f) \) as:

\[
E_1 \frac{d\lambda_2}{dA_c}(u_m - u_f) = \int_0^{\epsilon_m(A_c, \epsilon_f)} \frac{d\lambda_2^U}{dA_c}(u_m - u_f) dF(\epsilon_f, \epsilon_m) + \int_{\epsilon_m(A_c, \epsilon_f)}^\infty \frac{d\lambda_2^L}{dA_c}(u_m - u_f) dF(\epsilon_f, \epsilon_m)
\]

where \( F \) is the joint density of idiosyncratic productivity in the household.

From equation 8 it is easy to establish that the derivative \( \frac{d\lambda_2^U}{dA_c} \) is positive and the derivative \( \frac{d\lambda_2^L}{dA_c} \) is negative. In order to sign \( \frac{d\lambda_2}{dA_c}(u_m - u_f) \), in each relevant region of (48), we need to determine the difference in the welfare levels of husbands and wives. As it turns out, this difference is not of one sign. This is so because the limited commitment model has nothing to say about the absolute level of utility; it simply states that if ever participation is violated, a correction has to be made that makes one of the spouses as well off as if they were single. Since the model does not admit an analytical solution for the conditional expectation, we used numerical methods to compute the relevant integrals. Depending on the level of assets, we found that \( E_1[\frac{d\lambda_2}{dA_c}(u_m - u_f)] \) could be both positive or negative, which implies that the effect of limited commitment on household savings is

\[41\] Notice that the period one idiosyncratic productivity endowments are normalized to unity for both spouses.
ambiguous.\footnote{This result emerges also in Ligon et al. [2000].}

The more general case for $\gamma > 1$ yields similar results. For this model we can derive the following expression for the leading term in $46$:

\begin{equation}
E_1 \left[ \lambda_2 \frac{d u_m}{d A_c} + (1 - \lambda_2) \frac{d u_f}{d A_c} \right] = E_1 \left[ (A_c + \sum_g w(1 - \tau_N)\epsilon_g)^{-\gamma} \lambda_2 \left( \frac{1}{1 + f(\lambda_2, \epsilon)} \right)^{1 - \gamma} \left( \frac{\lambda_2}{w(1 - \tau_N)\epsilon_m^{(1 - \eta)(1 - \gamma)}} + \frac{f(\lambda_2, \epsilon)}{1 + f(\lambda_2, \epsilon)} \right)^{1 - \gamma} \left( \frac{1 - \lambda_2}{w(1 - \tau_N)\epsilon_f^{(1 - \eta)(1 - \gamma)}} \right) \right]
\end{equation}

The above expression suggests that the intrahousehold allocation affects the optimal savings of the family relative to the full commitment model, even beyond the term $E_1 \frac{dA_c}{dA_c} (u_m - u_f)$. To see how, first note that the bottom line of $49$ can be further simplified into:

\begin{equation}
((1 + f(\lambda_2, \epsilon))^{-\gamma} \left( \frac{\lambda_2}{w(1 - \tau_N)\epsilon_m^{(1 - \eta)(1 - \gamma)}} + \frac{f(\lambda_2, \epsilon)}{1 + f(\lambda_2, \epsilon)} \right)^{1 - \gamma} \left( \frac{1 - \lambda_2}{w(1 - \tau_N)\epsilon_f^{(1 - \eta)(1 - \gamma)}} \right) = (\lambda_2 \epsilon_m^{\omega} + (1 - \lambda_2)\epsilon_f^{\omega})^\gamma
\end{equation}

where $\omega = (1 - \eta)(1 - \gamma)/\gamma < 0$. Second, assume that male and female productivities in period 2 are perfectly negatively correlated, so that $\epsilon_m + \epsilon_f = \tau$ which is constant. It is obvious that $50$ is the only term that matters for household savings, as under these assumptions $(A_c + \sum_g w(1 - \tau_N)\epsilon_g)^{-\gamma} \lambda_2$ would be constant, no matter the realizations of $\epsilon_m$ and $\epsilon_f$.\footnote{Note that the covariance structure of wages within the household is extremely important for the sharing rule and, of course, the optimal allocation. In the two period model of this section, more negatively correlated shocks imply that the limited commitment problem in the household is more severe. If, on the other hand, shocks were perfectly correlated yielding $\epsilon_m = \epsilon_f$, then it is trivial to show that household rebargaining would not occur in equilibrium. The optimal weight $\lambda_2$ would equal a half, and the demand for savings of a two member household would be identical to the demand of a single earner household. When shocks are not perfectly correlated, the need to accumulate assets to buffer shocks to the labor income is less, and as a consequence couple households accumulate less wealth.} The term in $50$ exerts an influence to household savings because it changes the marginal benefit to the household. Since it is a concave function in $\epsilon_m$, higher uncertainty decreases the marginal utility, even under full commitment when the shares are constant. Moreover, if the shares $\lambda_2$ change with the endowment, as they do under limited commitment, they contribute further to the variability of $50$. This lowers the marginal gain from an extra unit of savings even further. In this example limited commitment means less rather than more savings. The analysis of the term $E_1 \frac{dA_c}{dA_c} (u_m - u_f)$ is similar with the log-log case and, for the sake of brevity, is omitted.

A.4 Competitive Equilibrium

In this section we briefly define the time invariant competitive equilibrium. Given a level of expenditure $G$ in the steady state, the tax schedule $\{\tau_K, \tau_W, \tau_C, \tau_{SS}\}$, social security policy and unintended bequests, the competitive equilibrium is a set of value functions $\{S_g, M, V_g\}$, household decision rules for consumption, savings and leisure, and measures of households over the state vector of assets, productivity, age, gender, marital status and the sharing rule $\lambda$ such that:

1. Given prices, $S_m$, $S_f$ and $M$ solve the functional equations and optimal policies derive. In particular, optimal policies are functions $c_{S,g}(a, X, j)$, $a'_{S,g}(a, X, j)$, $n_{S,g}(a, X, j)$ for consumption, assets and hours for singles and analogously $c_{M,g}(a, X, \lambda, j)$, $a'_{M}(a, X, \lambda, j)$, $n_{M,g}(a, X, \lambda, j)$ for couples.

2. Prices $w$ and $r$ satisfy:

$$w = (1 - \alpha)K^\alpha N^{-\alpha} \quad r = \alpha K^{\alpha - 1} N^{1 - \alpha} - \delta$$

where $N$ is the aggregate labor input in units of effective labor.
3. The social security policy satisfies:

\[
\nu N \tau_{SS} = \left( \sum_g \frac{1-\mu}{2} \int SS_g(\alpha, j) \Gamma_{S,g}(da \times dX \times \{j_R, \ldots, J\}) \right) \\
+ \mu \int SS_g(\alpha, j) \Gamma_M(da \times dX \times d\lambda \times \{j_R, \ldots, J\}))
\]

where \(\Gamma_{S,g}\) is the measure of bachelors over relevant states and \(\Gamma_M\) is the analogous object for married couples.

4. Accidental bequests satisfy:

\[
B = \frac{1}{\Phi_0} \left( \sum_g \frac{1-\mu}{2} \int a'_{S,g}(a, X, j)(1 - \psi_j) \Gamma_{S,g}(da \times dX \times \{1, \ldots, j_R - 1\}) \right) \\
+ \mu \int a'_{M}(a, X, \lambda, j)(1 - \psi_j) \Gamma_M(da \times dX \times d\lambda \times \{1, \ldots, j_R - 1\})
\]

5. The government budget constraint is balanced:

\[
G = \nu N \left( \sum_g \frac{1-\mu}{2} \int n_{S,g}(a, X, j)L_g(j)\epsilon_g\alpha_g \Gamma_{S,g}(da \times dX \times \{1, \ldots, j_R - 1\}) \right) \\
+ \sum_g \mu \int n_{M,g}(a, X, \lambda, j)\alpha_g\epsilon_g L_g(j) \Gamma_M(da \times dX \times d\lambda \times \{1, \ldots, j_R - 1\}) \\
+ \tau_C \left( \sum_g \frac{1-\mu}{2} \int c_{S,g}(a, X, j) \Gamma_{S,g}(da \times dX \times dj) \right) \\
+ \sum_g \mu \int c_{M,g}(a, X, \lambda, j) \Gamma_M(da \times dX \times d\lambda \times dj) \\
+ \tau_K \left( \sum_g \frac{1-\mu}{2} \int (ra + B) \Gamma_{S,g}(da \times dX \times dj) + \mu \int (ra + 2B) \Gamma_M(da \times dX \times d\lambda \times dj) \right)
\]

6. Market Clearing:

\[
K = \sum_g \int \frac{1-\mu}{2} a \Gamma_{S,g}(da \times dX \times dj) \quad + \mu \int a \Gamma_M(da \times dX \times d\lambda \times dj)
\]

\[
N = \sum_g \frac{1-\mu}{2} \int n_{S,g}(a, X, j)L_g(j)\epsilon_g\alpha_g \Gamma_{S,g}(da \times dX \times \{1, \ldots, j_R - 1\}) \\
+ \sum_g \mu \int n_{M,g}(a, X, \lambda, j)\alpha_g\epsilon_g L_g(j) \Gamma_M(da \times dX \times d\lambda \times \{1, \ldots, j_R - 1\})
\]

7. The measures \(\Gamma_{S,g}\) and \(\Gamma_M\) are consistent. In particular, for all subsets \(\mathcal{A}, \mathcal{X}, \Lambda, \mathcal{J}\) of the state space such that \(1 \notin \mathcal{J}\),

\[
\Gamma_{S,g}(\mathcal{A}, \mathcal{X}, \mathcal{J}) = \psi_j \int_{X' \in \mathcal{X}, a'_{S,g} \in \mathcal{A}, j+1 \in \mathcal{J}} \Gamma_{S,g}(da \times dX \times dj)
\]

\[
\Gamma_{M,g}(\mathcal{A}, \mathcal{X}, \Lambda, \mathcal{J}) = \psi_j \int_{X' \in \mathcal{X}, a'_{M,g} \in \mathcal{A}, \lambda' \in \Lambda, j+1 \in \mathcal{J}} \Gamma_{M,g}(da \times dX \times d\lambda \times dj)
\]
Figure 1: Responses of Capital, Output, Benefits and Returns to the Reform
Note: The figure plots the response of aggregate capital (top left), aggregate output (top right), retirement benefits (bottom left) and rate of returns (bottom right), where the solid line shows the gross return and the dashed line the return net of capital taxation.

Figure 2: Evolution of the Male Share
Note: The figure gives an example of the behavior of the male share over the life cycle. The top left shows the share in the initial steady state. The top right shows the change in the share that is due to shocks to the household’s labor income. The bottom panel compares the initial steady state (solid line), final (dashed line) and first period of the transition (crossed line).
Figure 3: Individual Consumption Uncertainty

Note: The figure plots changes in the variance of individual consumption relative to the original steady state. The solid line is the change in the final steady state. In order to measure the contribution of improved commitment, in the crossed line we represent the change in consumption volatility when we remove the influence of wealth but allow the household to be able to commit to the initial allocation as in the new steady state. For the sake of comparison the dashed line in the figure represents a constant $\lambda$ full commitment economy.
Figure 4: Volatility of the Sharing Rule

Note: The figure plots the standard deviation of $\lambda$ for various age groups, controlling for wealth. For each age we compute the share for each individual and for all next period realizations of the state vector. More precisely, we fix wealth and the initial value of $\lambda$ in a given period and let the household optimize in the next period, keeping track of all possible values of $\lambda$. In the solid line we represent the standard deviation of the sharing rule when the household is at the borrowing constraint. In the dashed line wealth is at the average of the low fixed effect families in the model. In the crossed line wealth is set equal to the average wealth in the economy.